

# AP<sup>®</sup> Precalculus

# Change in Tandem



We read a graph in mathematics from left to right,  
the same way we read a book.

EX #4: Consider the function  $y = g(x)$  shown at right,  
trace each interval where the function behavior is

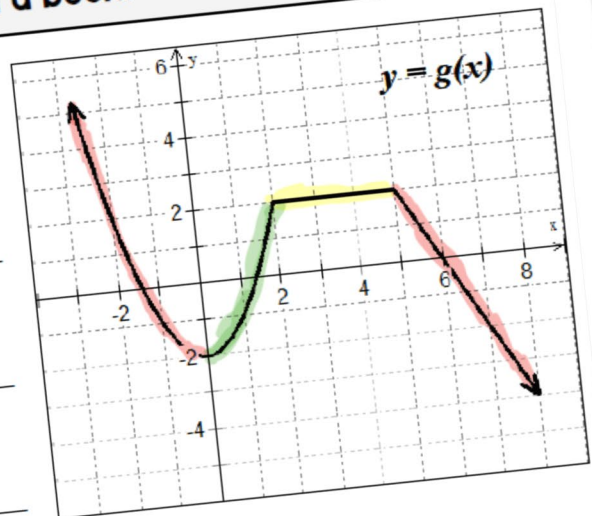
A. **Increasing**, using a GREEN pencil  
Identify the interval(s)  $0 < x < 2$

B. **Decreasing**, using a RED pencil  
Identify the interval(s)  $(-\infty, 0)$   $(5, \infty)$

C. **Constant**, using a YELLOW pencil  
Identify the interval(s)  $2 < x < 5$

D. State the **Domain** of the function  $(-\infty, \infty)$

E. State the **Range** of the function  $(-\infty, \infty)$



# Completely Editable

# 1.1

## Lesson I-A: Change in Tandem

### Function Rules

#### VOCABULARY

A **function** is a mathematical relation that maps a set of input values to a set of output values such that each input value is mapped to exactly one output value.

The set of input values is called the \_\_\_\_\_ of the function, and the set of output values is called the \_\_\_\_\_ of the function.

The "Rule of Four" concept, developed by the Harvard Calculus Consortium, will strengthen your understanding for different ways to represent functions.

### THE RULE OF FOUR (G.N.A.W.) FOUR WAYS TO REPRESENT A FUNCTION

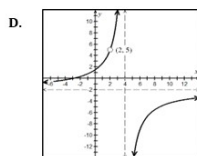
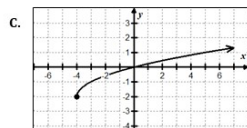
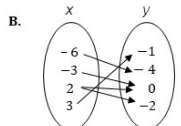
Graphically by points on a coordinate plane where input values are on the horizontal axis and output values are on the vertical axis

### Finding the Domain of a Function

If a function  $f$  is defined by an equation and no domain is specified, then the domain will be the largest set of real numbers for which the value of  $f(x)$  is a real number. Exclude any real numbers from a function's domain that cause division by zero or that result in an even root of a negative number; that is, an imaginary number.

EX #2: Find the domain and range in each of the relations below. Determine if the relation is a function.

A.  $\{(7, -2), (-3, 5), (1, 6), (3, -4), (2, 5)\}$



EX #3: Find the domain of each of the following relations. Is the relation a function? Use a graphing calculator to verify your findings.

A.  $f(x) = x^2 + 3x$

B.  $g(x) = \frac{x}{x^2 - 9}$

In general terms, we can say that a function **increases** over an open interval of its domain if its graph **rises from left to right on the interval**. Likewise, a function **decreases** over an open interval of its domain if its graph **falls from left to right on the interval**. Finally, a function is **constant** over an open interval of its domain if its graph is **horizontal on the interval**.

It is important to note that the **intervals** refer to the **x-values** where the **y-values** either **increase, decrease, or remain constant**.

### Definition of Increase, Decrease and Constant

Suppose that a function  $f$  is defined over an **open** interval,  $I$  and  $a$  and  $b$  are in  $I$ .

- ☐  $f$  is increasing on an interval  $I$  if, for any two points in the interval, a positive change in  $x$  results in a positive change in  $f(x)$ . That is, for all  $a$  and  $b$  in the interval, if  $a < b$ , then  $f(a) < f(b)$ .
- ☐  $f$  is decreasing on an interval  $I$  if, for any two points in the interval, a positive change in  $x$  results in a negative change in  $f(x)$ . That is, for all  $a$  and  $b$  in the interval, if  $a < b$ , then  $f(a) > f(b)$ .
- ☐  $f$  is constant on an interval  $I$  if, for any two points in the interval, a positive change in  $x$  results in a zero change in  $f(x)$ . For every  $a$  and  $b$ ,  $f(a) = f(b)$ .

### We read a graph in mathematics from left to right, the same way we read a book.

EX #4: Consider the function  $y = g(x)$  shown at right, trace each interval where the function behavior is

A. **Increasing**, using a GREEN pencil

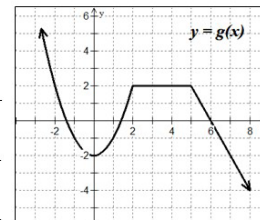
Identify the interval(s) \_\_\_\_\_

B. **Decreasing**, using a RED pencil

Identify the interval(s) \_\_\_\_\_

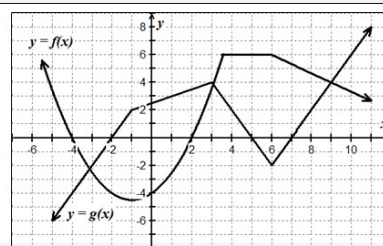
C. **Constant**, using a YELLOW pencil

Identify the interval(s) \_\_\_\_\_



### Comparing Functions Graphically

EX #11: Use the graphs of functions  $f(x)$  and  $g(x)$ , shown at right, to answer the following.

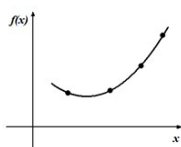


## Lesson I-B: Change in Tandem

### Concavity and Inflection Points

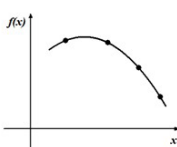
An **inflection point** is where a curve changes from **concave upward** to **concave downward** (or vice versa). We need Calculus to algebraically find where a curve goes from concave up to concave down.

#### Concave Up



Slope is increasing

#### Concave Down



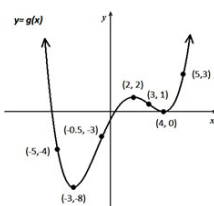
Slope is decreasing

EX #5: Use the graph of  $g(x)$  shown at right to determine intervals of concavity and state the points of inflection.

Concave Up: \_\_\_\_\_

Concave Down: \_\_\_\_\_

Points of Inflection: \_\_\_\_\_



### Intercepts

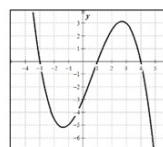
#### x - intercepts

The x-intercepts of the graph of  $f$ , if there are any, are the solutions of the equation  $f(x) = 0$ . That is:  $(x, 0)$

#### y - intercepts

If  $x = 0$  is in the domain of a function  $y = f(x)$ , then the y-intercept of the graph of  $f$  is the value of  $f$  at 0, or  $f(0)$ . That is:  $(0, y)$

EX #7: In the graph below, name the intercepts.



EX #8: Find the intercepts for the function algebraically. Given:

$$f(x) = x^3 - 3x^2 - 4x + 12$$

### Applications

EX #9: Tina is a sales-clerk at *Electronix Depot* who earns \$325 base pay plus \$18 for each item sold. Express her gross salary  $G$  as a function of the number  $x$  of items sold. Find her gross pay if she sells 24 items.

# NOTES

(1, -11)

## 1.1

Change in Tandem  
Practice Set 1

Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

**Problems 1 – 4,** Determine whether each relation represents a function. For each function, state the domain and range.

|    |     |    |   |   |   |
|----|-----|----|---|---|---|
| 1. | $x$ | 7  | 5 | 3 | 4 |
|    | $y$ | -3 | 0 | 1 | 3 |

2.  $3y - 6x^2 + 9 = 0$

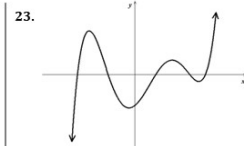
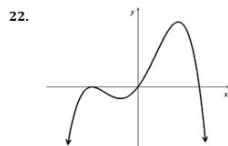
|    |                 |                 |
|----|-----------------|-----------------|
| 3. | <b>Employee</b> | <b>Password</b> |
|    | Justin          | 2587            |
|    | Emily           | 3041            |
|    | Ken             | 5206            |
|    | Nicole          | 7982            |

4.  $x^2 + y^2 = 9$

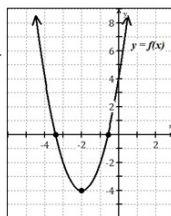
**Problem 20 – 21,** Complete the tables below for the function, given  $h(x) = f(g(x))$ 

|     |     |        |        |        |
|-----|-----|--------|--------|--------|
| 20. | $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|     | 0   | 1      | 5      |        |
|     | 1   | 0      | 3      |        |
|     | 2   | 4      | 2      |        |
|     | 3   | 2      | 1      |        |
|     | 4   | 5      | 3      |        |
|     | 5   | 3      | 0      |        |

|     |     |        |        |        |
|-----|-----|--------|--------|--------|
| 21. | $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|     | 0   |        | 4      | 1      |
|     | 1   |        | 1      | 5      |
|     | 2   |        | 0      | 2      |
|     | 3   |        | 5      | 0      |
|     | 4   |        | 3      | 4      |
|     | 5   |        | 2      | 3      |

**Problems 22 – 23,** Given the continuous function, determine how many inflection points **would be** found on each graph. Show the approximate location of each estimate.

## Free Response Question

Given  $f(x) = 2(x + 2)^2 - 4$ , answer each of the following:A. Find  $f(-5)$ . Explain the meaning of your result in relation to the graph.B. What is  $f(0)$ ? Does this value seem reasonable based on the graph of the function shown at right? Explain why or why not.C. The graph of  $f(x)$  has two real zeros. Notice that a zero of the function occurs between  $x = -4$  and  $x = -3$ . Use computations to justify why a zero of the function must exist at some  $x$ -value on this interval  $-4 \leq x \leq -3$ .D. Solve  $f(x) = -2$ , explain the meaning in relation to the graph of  $f$  shown above.**Problems 9 – 12,** Evaluate each function at the specified value of the independent variable and simplify.

9.  $f(x) = 2x^2 + 3x - 5$

A.  $f(-1)$

B.  $-f(x)$

C.  $f(2x)$

10.  $f(x) = |x| - 3$

A.  $f(-x)$

B.  $f(x - 1)$

C.  $f(3x)$

11.  $f(x) = \sqrt{x^2 - x}$

A.  $f(-1)$

B.  $f(-x)$

C.  $-f(x)$

12.  $f(x) = \frac{x}{x^2 - 1}$

A.  $-f(x)$

B.  $f(x + 1)$

C.  $f(x + h)$

## 1.1

Change in Tandem  
Practice Set 2

Name \_\_\_\_\_

Date \_\_\_\_\_ Period \_\_\_\_\_

**Problems 17 – 19,** The table below shows ordered pairs on the graph of a continuous function  $f(x)$ . Use the table to create a table of values for each function below that is a transformation of the graph of  $f(x)$ . State the shifts and/or reflections that occurs to  $f(x)$  in order to obtain the coordinates of the new graph.

|        |    |    |    |   |   |
|--------|----|----|----|---|---|
| $x$    | -3 | -1 | 1  | 3 | 5 |
| $f(x)$ | 4  | 2  | -5 | 1 | 3 |

17.  $g(x) = 2f(x - 3) + 4$

| Ordered Pairs on $f(x)$ | $x$ -coordinate on $g(x)$ | $y$ -coordinate on $g(x)$ | Ordered Pairs on $g(x)$ |
|-------------------------|---------------------------|---------------------------|-------------------------|
| (-3, 4)                 |                           |                           |                         |
| (-1, 2)                 |                           |                           |                         |
| (1, -5)                 |                           |                           |                         |
| (3, 1)                  |                           |                           |                         |
| (5, 3)                  |                           |                           |                         |

18.  $h(x) = -f(x + 2) - 3$

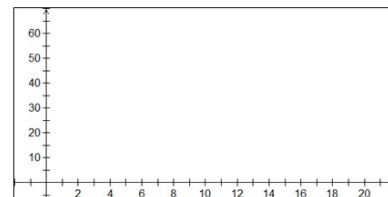
| Ordered Pairs on $f(x)$ | $x$ -coordinate on $h(x)$ | $y$ -coordinate on $h(x)$ | Ordered Pairs on $h(x)$ |
|-------------------------|---------------------------|---------------------------|-------------------------|
| (-3, 4)                 |                           |                           |                         |

**Problems 27 – 28,** Sketch a graph to represent each scenario below.

27. A car traveling at 45 mph accelerates to 60 mph in 8 seconds. It maintains that speed for the next 5 seconds, and then slows to a stop during the next 7 seconds.

- A. Draw a graph to represent the vehicle behavior as a function of time. Label the axes properly.
- B. State the domain and range of the scenario.

C. Write a linear equation that represents the behavior of the car on the interval from 8 seconds to 13 seconds.



28. Ethan leaves home to walk to school, walks 2 blocks in 4 minutes at a constant speed. He forgot to bring his lunch, so he runs home in 3 minutes. It takes him two minutes to find his lunch and leave. Ethan walks the 5 blocks to school in 12 minutes and stops at the front of the school to talk to his friend Matt for 4 minutes.

- A) Draw a graph of Ethan's distance from home (in blocks) as a function of time. Label the axes properly.
- B) State the domain and range of the scenario.

## Practice Sets

F. In general, explain the difference in the meaning when evaluating  $f(a)$ , and solving the equation  $f(x) = a$ .