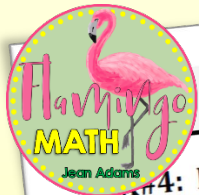


AP[®] Precalculus

Change in Arithmetic & Geometric Sequences



Analyzing Arithmetic Sequences

EX #4: Is the sequence an arithmetic sequence? If so, write a general rule. 1, 5, 9, 13, 17, ...

common difference is 4

$$a_0 = 1 - 4$$
$$a_0 = -3$$

sequence $a_n = 4n - 3$

EX #5: Find the 46th term of the sequence 2, 5, 8, 11, 14, ...

$$d = 3$$
$$a_0 = -1$$
$$a_n = 3n - 1$$

$$a_{46} = 3(46) - 1$$
$$= 138 - 1$$
$$= \underline{\underline{137}}$$

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2.1

Lesson I-A: Change in Arithmetic and Geometric Sequences

When you think of a "sequence of events" you likely would consider what happened first, second, and so on. In mathematics, the word *sequence* is also used in the same way, some list of numbers has a first element, a second element, a third element, and so on. One famous example, known as the **Fibonacci sequence** is found throughout nature: 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Can you discover the rule for the next term?

DEFINITION OF A SEQUENCE

An **infinite sequence** is a function whose domain is the set of positive integers. The function values are the **terms** of the sequence.

$$a_1, a_2, a_3, a_4, \dots, a_n, \dots$$

If the domain of the function consists of the first n positive integers only, the sequence is a **finite sequence**.

- ☐ **Nth term** – the general term. The letter k is sometimes used instead of n .
- ☐ **Finite Sequence** has a certain number of terms.
- ☐ **Term of a Sequence** – each number in a sequence.

Generating a Sequence Using an Explicit Formula

EX #1: A sequence has an explicit formula $a_n = 12n + 3$. Find term a_{12}

Arithmetic Sequences

Arithmetic sequences have the same differences between terms (add):

$$d = a_2 - a_1, a_3 - a_2, \dots$$

d = common difference between consecutive terms

EXPLICIT

or

GENERAL FORM:

$$a_n = a_k + d(n - k)$$

For $n \geq 1$,

where a_k is the k th term

$$a_n = dn + a_0$$

$$a_0 = a_1 - d$$

Analyzing Arithmetic Sequences

EX#4: Is the sequence an arithmetic sequence? If so, write a general rule. 1, 5, 9, 13, 17, ...

Finding General Terms

EX #7: The numbers in the sequences below follow a general pattern. Can you write the general rule?

A. 1, 1.25, 1.5, 1.75, ...

B. 4, -1, -6, -11, -16, ...

Using Explicit Formulas

EX #8: The number of seats in the first 16 rows of an arena form an arithmetic sequence. If there are 20 seats in Row 1, 23 seats in Row 2, how many seats are in Row 16?

Lesson I-B: Change in Arithmetic and Geometric Sequences

Geometric Sequences

A **geometric sequence** is any sequence with a constant ratio between consecutive terms. With a starting value " a " and common ratio " r " is in the form $a, ar, ar^2, ar^3, \dots, ar^n$.

EX. 4, 8, 16, 32, 64, ... $4(2)^n$

General Form and Common Ratios

The n th term (general term) of a geometric sequence with first term a_1 and common ratio r is

$$a_n = a_1 r^{n-1}$$

Geometric sequences have common ratios between consecutive terms.

$$\text{Common ratio: } r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}$$

Each term in a geometric sequence can be obtained recursively from its previous term by multiplying by r :

Formula:

General Form:

$$g_n = g_0 r^n$$

$$a_n = a_{n-k} r^{n-k}$$

Finding Terms of a Geometric Sequence

EX #10: Find the indicated terms of the geometric sequences.

A. Find a_6 and a_n for the geometric sequence
3, -15, 75, ...

B. Determine r and a_1 for the geometric sequence
with $a_4 = -8$ and $a_{11} = \frac{1}{16}$

EX #11: When a radioactive substance decays, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table shows the amount of an unknown substance, initially and after 2 hours. Find the amounts left after 1 hour, 3 hours, and 4 hours.

Hours Lapsed	0	1	2	3	4
Grams of radioactive substance	1320		384		

EX #13: A Christmas tree farm has 10,000 trees on its property in 2022. Each year after 10% of the trees are harvested and 700 new saplings are planted.

A. Write a general formula t_n that gives the current number of trees on the farm in the year n , where $n = 0$ corresponds to 2022.

B. Use the formula from part (a) to find the number of trees for $n = 1, 2, 3, 4$. Interpret these values in the context of the situation.

EX #14: A ball is dropped from a height of 30 feet. Each time it hits the ground, the ball bounces up to 0.8 of the previous height.

A. Complete the table below to model the data in the scenario.

n	1	2	3	4	5	6	7	8
height	30							

B. How high will the ball bounce up to after it strikes the ground for the n^{th} time?

Guided Notes

2.1

Change in Arithmetic & Geometric Sequences Practice Set 1

Name _____

Date _____ Period _____

Problems 1 – 4, determine whether or not the sequence is arithmetic. If it is, find the common difference.

1. $1, 1, 2, 3, 5, 8, \dots$

2. $-83, -74, -65, -56, \dots$

3. $4, \frac{5}{2}, 1, \frac{-1}{2}, \dots$

4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

Problems 5 – 8, write the first five terms of each sequence.

5. $a_n = 7 + 5n$

6. $a_n = 2^{n-1}$

7. $a_n = 120 - 8n$

8. $a_n = \frac{n}{n+2}$

19. How many logs will be in a pile of timbered trees if there are 30 logs in the bottom layer, 29 in the second, and so on until there is one in the top layer?

20. Each school year, Mu Alpha Theta plans to add 5 more members to their tutoring team for helping math students who struggle. This year

A. Write a general formula for the number of

B. Write the first five terms of the sequence.

C. How many tutors will the club have in the 1

Problems 13 – 16, identify each sequence as arithmetic or geometric. Then generate the first five terms.

13. $1, -2, 4, -8, \dots$

15. $-4, 12, -36, 108, \dots$

Problems 17 – 20, find the missing terms of each sequence.

17. $2187, \square, \square, \square, 27, \dots$

19. $0.1024, \square, \square, \square, 4, \dots$

Problems 21 – 24, for the geometric sequence 2,

Problems 13 – 16, write the general rule for the sequence and find the 14th term.

13. $a_{13} = 15, d = 4$

14. $a_{13} = 15, d = -7$

15. $a_{15} = 15, d = -11$

16. $a_{15} = 15, d = 19$

Problems 17 – 21, solve.

17. You know your school bus stops at 2-minute intervals. The first stop of the route begins at 6:20 A.M. every day. If your stop is the 18th stop, when will the bus arrive for you?

2.1

Change in Arithmetic & Geometric Sequences Practice Set 2

Name _____

Date _____ Period _____

Problems 1 – 4, is the given sequence geometric? If so, identify the common ratio and find the next two terms.

1. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

2. $18, -6, 2, -\frac{2}{3}, \dots$

3. $9, 0.9, 0.09, 0.009, \dots$

4. $0.5, 0.25, 0.125, 0.0625, \dots$

Problems 5 – 8, for each sequence. Then generate the first five terms.

6. $a_1 = 0.0175, r = 5$

Problems 29-30, solve.



29. Caliber One Construction purchased a new service vehicle for \$58,200. Each year they will depreciate the value of the vehicle for tax purposes by 15%.

A. Write a general formula for the value of the vehicle.

B. What will be the value of the vehicle after 5 years?

C. When will the vehicles value be less than \$20,000?

30. Andrea deposited \$500 at the beginning of each month. The balance, A , after n months is given by the formula:

$$A = 500 \left(1 + \frac{0.085}{12} \right)^n + \dots + 500$$

A. Write a general formula that represents Andrea's balance after n months based on this rate of return.

B. How long will it take before the value of the account reaches \$100,000?

C. What will be the value of the account in 30 years?

Problems 25 – 26, write the explicit formula for the sequence. Then generate the first five terms.

25. $a_1 = 5, r = -2$

26. $a_1 = 729, r = -\frac{1}{3}$

Problems 27 – 28, solve.

27. Jordan got some helium balloons from her best friend Madison on her birthday. The balloons lose one-fifth of the helium each day. Each balloon was filled to a volume of 2500 in³.

A. Write the geometric sequence representing the amount of helium in the balloons for the first five days.

B. What is the common ratio of the sequence? [Hint: Volume is in cubic inches.]

C. How much helium will be in the balloons on the 10th day?



Practice Sets

2.1 Change in Arithmetic and Geometric Sequences Set 1 **Form A**

Name _____
Date _____ Period _____

1. Find the 23rd term of the sequence.
7, 4, 1, -2, -5, ...

2. Write the general rule and find the 12th term.
 $a_{17} = 28, d = 5$

Use the following information for problems 3 and 4. Your parents give you a gift of \$250 for your birthday. You plan on adding \$20 each week.

3. Write the amount in the account after each deposit as an arithmetic sequence, let $a_1 = 250$.

4. How much money will you have on your next birthday, 52 weeks away?

Grade: _____

2.1 Change in Arithmetic and Geometric Sequences Set 1 **Form B**

Name _____
Date _____ Period _____

1. Find the 21st term of the sequence.
-3, 1, 5, 9, 13, ...

2. Write the general rule and find the 15th term.
 $a_{12} = 42, d = -3$

Use the following information for problems 3 and 4. Your parents give you a gift of \$200 for your birthday. You plan on adding \$25 each week.

3. Write the amount in the account after each deposit as an arithmetic sequence, let $a_1 = 200$.

4. How much money will you have on your next birthday, 52 weeks away?

Grade: _____

2.1 Change in Arithmetic and Geometric Sequences Set 2 **Form A**

Name _____
Date _____ Period _____

For 1-3, identify each sequence as arithmetic, geometric, or neither. Then find the next two terms.

1. 3, 7, 11, 15, ...

2. 64, -32, 16, -8, ...

3. 4, 6, 9, 13, ...

4. Write the explicit formula and find the 6th term. $a_1 = 5, r = -3$.

5. John deposited \$7000 at the beginning of the year into a mutual fund that pays 4.5% interest compounded monthly. The balance, A , at the end of n months is given by

Grade: _____

2.1 Change in Arithmetic and Geometric Sequences Set 2 **Form B**

Name _____
Date _____ Period _____

For 1-3, identify each sequence as arithmetic, geometric, or neither. Then find the next two terms.

1. 5, 8, 12, 17, ...

2. 12, 9, 6, 3, 0, ...

3. -729, 243, -81, 27, ...

4. Write the explicit formula and find the 7th term. $a_1 = 4, r = -2$

5. Tom deposited \$7500 at the beginning of the year into a mutual fund that pays 4.3% interest compounded monthly. The balance, A , at the end of n months is given by

Grade: _____

Daily Lesson Quiz