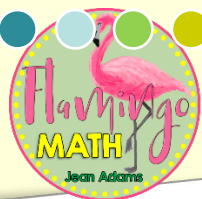


AP[®] Precalculus

Change in Linear & Exponential Functions



Generating an Exponential Function from a Geometric Sequence

EX #8: Use the graph of the sequence shown at right to

A. Complete the table below.

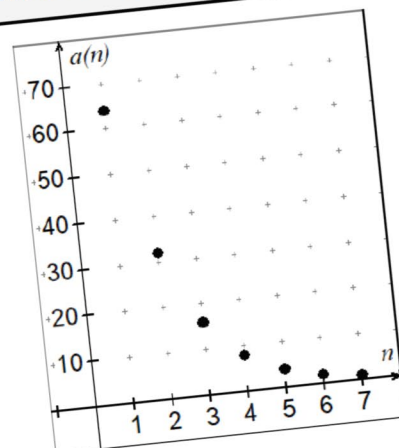
n	1	2	3	4	5	6	7
a_n	64	32	16	8	4	2	1

B. Find the common ratio r and write an explicit rule, a_n .

$$r = 2$$
$$a_n = 64(2)^{n-1}$$
$$a_n = 128(2)^n$$

C. Use the information from part (B) to write an exponential function that corresponds to the n th term expression.

$$f(n) = 64(2)^{n-1} \quad \text{or} \quad f(n) = 128(2)^n$$



Completely Editable

2.2

Lesson 2-A: Change in Linear and Exponential Functions

Let's look at the relationships between linear functions and arithmetic sequences. We know that a linear function of the form $y = mx + b$ describes a continuous function whose domain and range are all real numbers. The y -intercept is the point $(0, b)$ and the rate of change of the function is the value of m which represents the slope of the line over its domain. Arithmetic sequences can be represented by linear functions. However, sequences and their corresponding functions may have different domains.

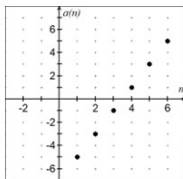
Generating a Linear Function from an Arithmetic Sequence

EX #1: Use the graph of the sequence shown at right to

A. Complete the table below.

n	0	1	2	3	4	5	6
a_n		-5					

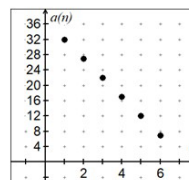
B. Find the difference d and write an explicit rule, a_n .



Arithmetic Sequences and Function Notation

The **explicit formula** denotes the n th term of the sequence where n tells the term's location. It defines the sequence as a formula in terms of n . It may be written in either **subscript notation** a_n or by using **function notation** $f(n)$.

EX #4: Given the sequence $\{32, 27, 22, 17, 12, 7, \dots\}$ Find an explicit formula in both subscript notation and function notation.



Subscript Notation:

Term Number	Term	Subscript Notation	Function Notation
1	32	a_1	$f(1)$
2			
3			
4			
5			
6			

Using Explicit Formulas

EX #6: Elizabeth waters her vegetable garden using water she collects in a 55-gallon rain barrel. On May 3, the barrel was full of water, and on May 8 the barrel contained 47 gallons of water.

A. Write the first six terms of the sequence.

B. Write an explicit rule using both subscript notation and function notation.

C. Explain when the rain barrel will be empty, assume there is no rain in the next 10-day period.

EX #7: Allan is assigned a book to read for his biology class. Counting today, Monday, as the first day, he finds that he must read through page 385 by day 12, and through page 665 by day 20.

Lesson 2-B: Change in Linear and Exponential Functions

We have seen that when the output values of a function change at a constant rate, the function is linear. Now let's look at what happens when the output values change proportionally. When your sequence is geometric, the rate of change will increase (or decrease) as the value of n increases.

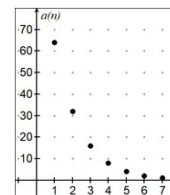
Generating an Exponential Function from a Geometric Sequence

EX #8: Use the graph of the sequence shown at right to

A. Complete the table below.

n	1	2	3	4	5	6	7
a_n	64	32			4		

B. Find the common ratio r and write an explicit rule, a_n .

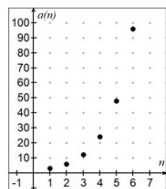


C. Use the information from part (B) to write an exponential function that corresponds to the n th term expression.

Geometric Sequences and Function Notation

The **explicit formula for the n th term** of the geometric sequence, $g_n = g_0 r^n$ can be modeled by the exponential function $f(x) = ab^x$. Similarly, the k th term formula, $g_n = g_k r^{n-k}$ will graphically correspond to $f(x) = y_1 r^{(x-x_1)}$ based on a known ratio r , and a point (x_1, y_1) .

EX #11: Given the sequence $\{3, 6, 12, 24, 48, 96, \dots\}$ Find an explicit formula in both subscript notation and function notation.



Subscript Notation:

Function Notation:

Term Number	Term	Subscript Notation	Function Notation
1	3	a_1	$f(1)$
2			
3			
4			
5			
6			
n	$\vdots$		

EX #12: Given the explicit formula for the sequence $a(n) = 6\left(\frac{1}{2}\right)^{n-1}$

A. Write the first 6 terms of the sequence.

EX #13: Tim released a helium-filled balloon that rose 200 feet in the first minute. In each succeeding minute, the balloon rises only 50% as far as it rose in the previous minute.

A. Write a general formula $a(n)$ that gives the height of the balloon for time in the air:

B. Use the formula from part (a) to find the height the balloon reaches after 8 minutes.

n	1	2	3	4	5	6	7	8
height	200							

EX #14: Big Tech Electronics advertises a monthly payment plan for the purchase of the best-selling tablet. Buyers pay \$50 at the end of the first month, \$55 at the end of the second month, \$60.50 at the end of the third month, and so on for one year:

A. What will the payments be at the end of the 4th, 6th, and 10th months?

B. How much will a customer pay for the tablet with this payment plan?

Guided Notes

2.2

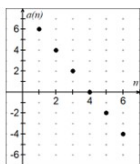
Change in Linear & Exponential Functions
Practice Set 1

Name _____

Date _____ Period _____

Problems 1–2, use the graph of the sequence to complete the table. Find the difference and write an explicit rule for a linear function that corresponds to the n th term expression.

1.



n	0	1	2	3	4	5	6
a_n		6					

2.



n	0	1	2	3	4	5	6
a_n		-3					

10. On a warm summer day in Colorado, a parcel of air rises over the Rocky Mountains. It cools at the dry adiabatic lapse rate of 5.5°F per 1000 feet until it reaches its dew point. If the ground temperature is 76°F ,

A. Write an explicit formula for the sequence of temperatures, $\{T(n)\}$, for the parcel of air that has risen n thousand feet.

B. Use your function to find the temperature of the air if it has risen 12,000 feet. Support your answer with work.

11. Cameron borrowed money interest-free to pay for his boat repairs. He is repaying the loan to the marina in equal monthly payments.

Payment #	n	1	2	3	4
Loan Balance	$f(n)$	960	835	710	585

A. Write an explicit formula for the sequence of loan balances.

Problems 5–8, find the common difference and write the general rule for the sequence in function notation. Find the total number of terms in the sequence,

5. $\{8, 1, -6, \dots, -48\}$

6. $\{-8, -2.75, 2.5, \dots, 55\}$

7. $\{1.8, -1.1, -4, \dots, -64.9\}$

8. $\{4.1, 3.7, 3.3, \dots, -15.5\}$

Problems 9–12, solve.

9. The graph at right shows how the cost of a hiking expedition depends on the number of hikers.



2.2

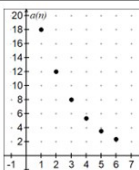
Change in Linear & Exponential Functions
Practice Set 2

Name _____

Date _____ Period _____

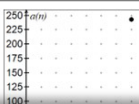
Problems 1–2, use the graph of the sequence to complete the table. Find the ratio and write an explicit rule for an exponential function that corresponds to the n th term expression.

1.



n	1	2	3	4	5	6
a_n	18	12			$\frac{32}{9}$	

2.



n	1	2	3	4	5	6	7
a_n	$\frac{1}{3}$	1			27		

Problems 5–8, find the ratio and the a_0 term. Then, write the general rule, $f(n) = ab^n$ for the sequence. Find the total number of terms in the sequence,



5. $\{-2, -8, -32, \dots, -8192\}$

6. $\{3125, 1250, 500, \dots, 0.8192\}$

7. $12, 3\frac{3}{4}, \frac{3}{16}, \dots$

8. $\{42, 126, 378, \dots, 275562\}$

Problems 9–12, solve.

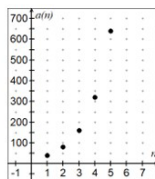
9. The graph at right represents a chocolate factory during Valentine week. Workers packed boxes of candy over the 7-day period.

A. Represent the sequence in a table.

n	1	2	3	4	5	6	7
$f(n)$	40	80			640		

B. Is the sequence geometric? Explain.

C. Write an explicit rule $f(n)$ for the sequence of candy boxes packed. Show your work to support your answer.



10. Anna dropped a marble into a cylinder of gelatin solution that was 24 inches deep. The marble began to sink slowly. In each second, the marble descended 15% lower than the previous second. The pattern for $n = 0, 1, 2$ creates the sequence $\{24, 20.4, 17.34, \dots\}$

A. Write a general formula $f(n) = ab^n$ that gives the height of the marble from the bottom of the cylinder for time in the gelatin.

B. Use the formula from part (a) to find the depth the marble sinks after 10 seconds.

C. At what time will the marble be 2 inches from the bottom of the cylinder?

11. A culture contains 10,000 bacteria initially and increases by 20% every hour.

A. Find a formula for the number $N(t)$ of bacteria present after t hours.

B. How many bacteria are in the culture after 10 hours?

12. A rubber ball is dropped from a height of 25 feet. The ball bounces to 80% of its previous height with each bounce.

A. Set up a model in the table below to represent each "bounce" starting with the height of the first bounce, listing 5 bounces. Round answers to the nearest tenth of a foot.

n	1	2	3	4	5

D. When will the ball bounce less than 1 foot high?

Practice Sets

2.2 Changes in Linear & Exponential Form A Functions Set 1Name _____
Date _____ Period _____Use the following sequence for 1 and 2.
{12, 11.6, 11.2, ... 6.4}

1. Find the common difference and write the general rule in function notation.

2. Find the number of terms in the sequence.

Use the following table for 3 and 4.

# Guests	1	2	3	4
Cost	\$250	\$350	\$450	\$550

3. Why is the sequence arithmetic?

4. Write the explicit formula in function notation and find the cost for 6 guests.

Grade: _____

2.2 Changes in Linear & Exponential Form B Functions Set 1Name _____
Date _____ Period _____Use the following sequence for 1 and 2.
{8, 7.4, 6.8, ... 2.6}

1. Find the common difference and write the general rule in function notation.

2. Find the number of terms in the sequence.

Use the following table for 3 and 4.

# Guests	1	2	3	4
Cost	\$225	\$300	\$375	\$450

3. Why is the sequence arithmetic?

4. Write the explicit formula in function notation and find the cost for 6 guests.

Grade: _____

2.2 Changes in Linear & Exponential Form A Functions Set 2Name _____
Date _____ Period _____1. Given the geometric sequence where $a_2 = 8$, $r = 4$, write an exponential function that will contain the domain of the sequence, and find the 5th term.Use the following information for 2 - 4.
A company buys a piece of equipment for \$60,000. The equipment loses (depreciates) 20% of its value each year.2. Write a general formula $f(n) = ab^n$ that gives the value of the equipment after n years.

3. What would be the value after 4 years?

4. After how many years will the value be less than \$15,000?

Grade: _____

2.2 Changes in Linear & Exponential Form B Functions Set 2Name _____
Date _____ Period _____1. Given the geometric sequence where $a_2 = 9$, $r = 3$, write an exponential function that will contain the domain of the sequence, and find the 6th term.Use the following information for 2 - 4.
A company buys a piece of equipment for \$80,000. The equipment loses (depreciates) 20% of its value each year.2. Write a general formula $f(n) = ab^n$ that gives the value of the equipment after n years.

3. What would be the value after 5 years?

4. After how many years will the value be less than \$20,000?

Grade: _____

Daily Lesson Quiz