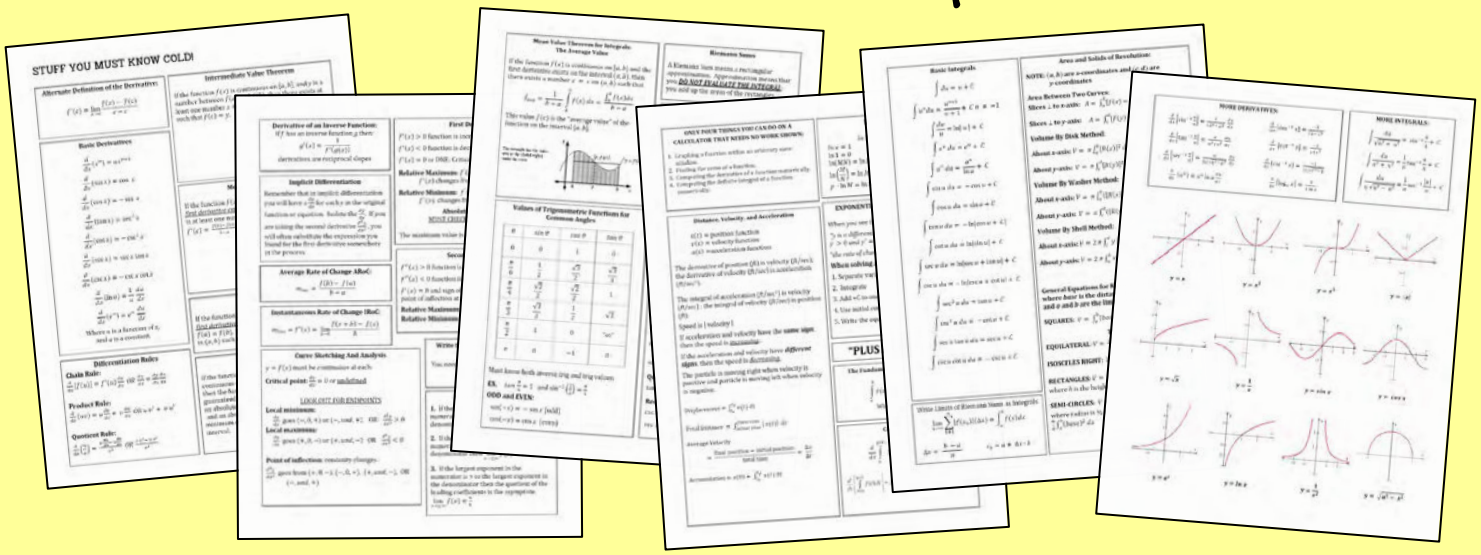
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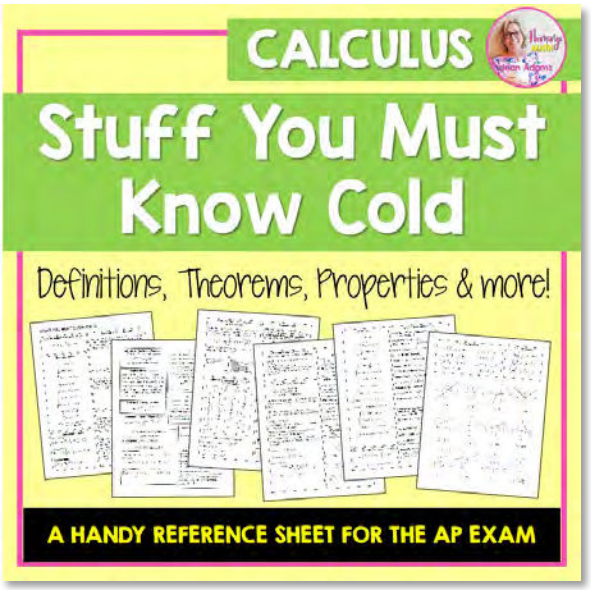


Stuff You Must Know Cold

Definitions, Theorems, Properties & more!



A HANDY REFERENCE SHEET FOR THE AP EXAM



AP CALCULUS
STUFF YOU MUST KNOW COLD

This is a handy reference sheet to help students prepare for the AP Calculus Exam

Grab this great **STUFF YOU MUST KNOW COLD** handout. It is an adaptation from Sean Bird's material at Covenant Christian School, used with permission.

There are six pages of everything important for the **AP CALCULUS AB STUDENT** to know backwards and forwards...definitions, theorems, properties, family of functions and more.

It's a handy reference sheet. Get your students ready from the beginning of school. There is also a BLANK SET where students can complete the information from memory.

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STUFF YOU MUST KNOW COLD!

Alternate Definition of the Derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

Where u is a function of x, and a is a constant.

Differentiation Rules

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx} \; OR \; \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule:

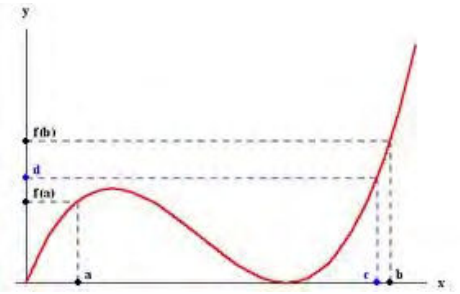
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \; OR \; u\,v' + v\,u'$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \; OR \; \frac{v\,u' - u\,v'}{v^2}$$

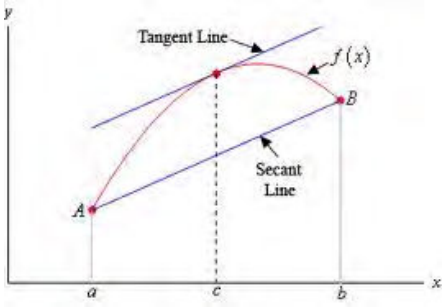
Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = y$.



Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *the first derivative exists on the interval* (a, b) then there is at least one number $x = c$ in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

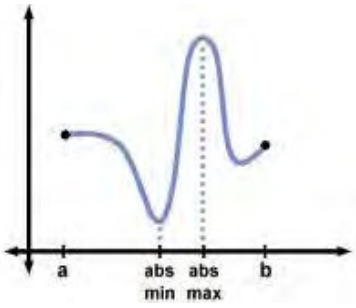


Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *the first derivative exists on the interval* (a, b) **AND** $f(a) = f(b)$, then there is at least one number $x = c$ in (a, b) such that $f'(c) = 0$.

Extreme Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, then the function is guaranteed to have an absolute maximum and an absolute minimum on the interval.



Derivative of an Inverse Function:

If f has an inverse function g then:

$$g'(x) = \frac{1}{f'(g(x))}$$

derivatives are reciprocal slopes

Implicit Differentiation

Remember that in implicit differentiation you will have a $\frac{dy}{dx}$ for each y in the original function or equation. Isolate the $\frac{dy}{dx}$. If you are taking the second derivative $\frac{d^2y}{dx^2}$, you will often substitute the expression you found for the first derivative somewhere in the process.

Average Rate of Change ARoC:

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change IRoC:

$$m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

First Derivative:

$f'(x) > 0$ function is increasing.

$f'(x) < 0$ function is decreasing.

$f'(x) = 0$ or DNE: Critical Values at x.

Relative Maximum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $+$ to $-$.

Relative Minimum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $-$ to $+$.

Absolute Max or Min:

MUST CHECK ENDPOINTS ALSO

The maximum value is a y-value.

Second Derivative:

$f''(x) > 0$ function is concave up.

$f''(x) < 0$ function is concave down.

$f'(x) = 0$ and sign of $f''(x)$ changes, then there is a point of inflection at x.

Relative Maximum: $f''(x) < 0$

Relative Minimum: $f''(x) > 0$

Curve Sketching And Analysis

$y = f(x)$ must be continuous at each:

Critical point: $\frac{dy}{dx} = 0$ or undefined

LOOK OUT FOR ENDPOINTS

Local minimum:

$\frac{dy}{dx}$ goes $(-, 0, +)$ or $(-, und, +)$ OR $\frac{d^2y}{dx^2} > 0$

Local maximum:

$\frac{dy}{dx}$ goes $(+, 0, -)$ or $(+, und, -)$ OR $\frac{d^2y}{dx^2} < 0$

Point of inflection: concavity changes

$\frac{d^2y}{dx^2}$ goes from $(+, 0, -)$, $(-, 0, +)$, $(+, und, -)$, OR $(-, und, +)$

Write the equation of a tangent line at a point:

You need a slope (derivative) and a point.

$$y_2 - y_1 = m(x_2 - x_1)$$

Horizontal Asymptotes:

1. If the largest exponent in the numerator is < largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = 0$.

2. If the largest exponent in the numerator is > the largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = DNE$

3. If the largest exponent in the numerator is = to the largest exponent in the denominator then the quotient of the leading coefficients is the asymptote.

$\lim_{x \to \pm\infty} f(x) = \frac{a}{b}$

ONLY FOUR THINGS YOU CAN DO ON A CALCULATOR THAT NEEDS NO WORK SHOWN:

1. Graphing a function within an arbitrary view window.
2. Finding the zeros of a function.
3. Computing the derivative of a function numerically.
4. Computing the definite integral of a function numerically.

LOGARITHMS

Definition:

$$ln\ N = p \leftrightarrow e^p = N$$

$ln\ e = 1$

$\ln 1 = 0$

$\ln(MN) = \ln M + \ln N$

$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$

$p \cdot \ln M = \ln M^p$

Distance, Velocity, and Acceleration

$s(t) =$ position function
$v(t) =$ velocity function
$a(t) =$acceleration function

The derivative of position (ft) is velocity (ft/sec); the derivative of velocity (ft/sec) is acceleration (ft/sec^2).

The integral of acceleration (ft/sec^2) is velocity (ft/sec) ; the integral of velocity (ft/sec) is position (ft).

Speed is | velocity |

If acceleration and velocity have the ***same sign***, then the speed is *increasing*..

If the acceleration and velocity have ***different signs***, then the speed is *decreasing.*

The particle is moving right when velocity is positive and particle is moving left when velocity is negative.

Displacement $= \int_{t_0}^{t_f} v(t)\ dt$

Total Distance $= \int_{initial\ time}^{final\ time} |\ v(t)|\ dt$

Average Velocity

$$= \frac{\text{final position} - \text{initial position}}{\text{total time}} = \frac{\Delta x}{\Delta t}$$

Accumulation $= x(0) + \int_{t_0}^{t_f} v(t)\ dt$

EXPONENTIAL GROWTH and DECAY:

When you see these words use: $y = Ce^{kt}$

"y is a differentiable function of t such that $y > 0$ *and* $y' = ky$ *"*

"*the rate of change of y is proportional to y*"

When solving a differential equation:

1. Separate variables first
2. Integrate
3. Add +C to one side
4. Use initial conditions to find "C"
5. Write the equation if the form of $y = f(x)$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where $F'(x) = f(x)$

Corollary to FTC

$$\frac{d}{dx}\int_a^{g(u)} f(t)dt = f\big(g(u)\big)\frac{du}{dx}$$

$$\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)\,dt\right] = f\left(g\left(x\right)\right)\bullet g'\left(x\right) - f\left(h\left(x\right)\right)\bullet h'\left(x\right)$$