

AP[®] Calculus

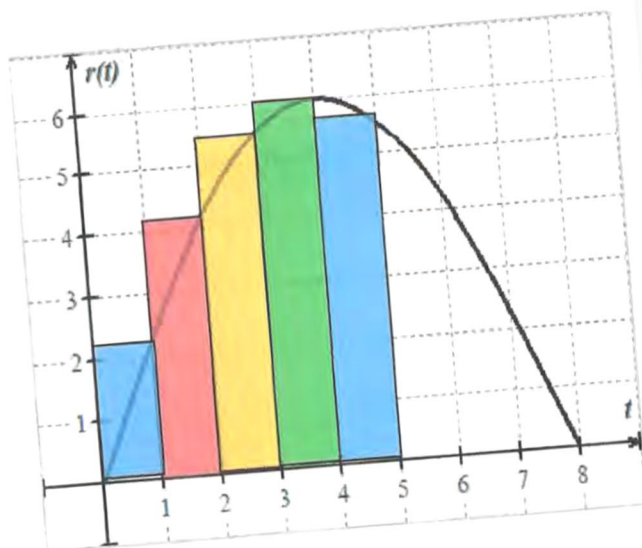
Approximating Area with Riemann Sums



Consider a 30-gallon aquarium is being filled with water at a rate modeled by $r(t) = 6 \sin\left(\frac{\pi}{8}\right)t$, where t is time in minutes, and $r(t)$ is gallons per minute. Find the volume of water in the tank after 5 minutes by using five rectangles that are 1-minute wide and $r(t)$ in height.

t	$r(t)$	Area Total
1	2.296	2.296
2	4.243	6.539
3	5.543	12.082
4	6.000	18.082
5	5.543	23.625

After 5 minutes, the aquarium has 23.625 gallons in the



Completely Editable

6.1

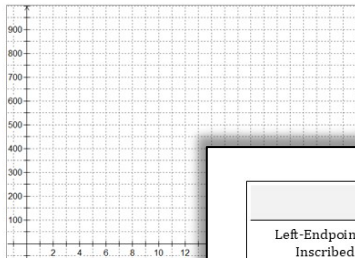
Lesson 6.1: Approximating Areas with Riemann Sums

Topic 6.1: Exploring Accumulations of Change

Let's take a look at the relationship between the graph of a *rate of change function* and the area of the region between that derivative graph and the x -axis. Consider the tropical cyclone *Imelda* that brought more than 40 inches of rainfall over a 72-hour period in Southeast Texas during September 2019.

EX #1: In order to avoid flood water entering your home, you rent a pump that can remove water at a rate of 500 gallons per hour for the first 8 hours of pumping. As the storm grew stronger, you got a second pump; and, for the next 16 hours you could remove 900 gallons per hour using two pumps.

- 1A. Draw a graph that models the removal of storm water from your property. Label the axes correctly.



- 1B. When we say "area under a curve" this represents the area between the curve and the horizontal axis. Find the area under the graph that represents in context to the tropical storm *Imelda*.

- 1C. Find the total area under the graph for the entire context to the scenario.

Finding Area Using Riemann Sums

Topic 6.2: Approximating Areas With Riemann Sums

What approach can we take if the rate of change function is not constant?

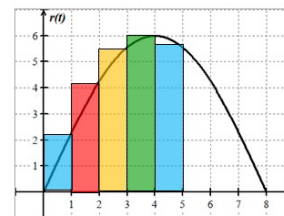
The two big ideas in calculus are the tangent line problem and the area problem. In the *tangent line problem*, you saw how the limit process could be applied to find the slope of the tangent line (i.e. the derivative) at any point on the curve. A second classic problem in calculus is in finding the *area under a curve* that is bounded by the graph, the x -axis, and any two x -values.

A basic overview of "areas as limits." In the "limit of rectangles" approach, we take the area under a curve $y = f(x)$ above the interval $[a, b]$ by approximating a collection of *inscribed* or *circumscribed* rectangles in such a way that, the more rectangles used, the better the approximation. Finally, the number of rectangles is increased without limit and, bingo, we get the area! Now known as integration.

Specifically, we are interested in finding the area A of a region bounded by the x -axis, the graph of a *nonnegative* function $y = f(x)$ defined on some interval $[a, b]$.

Note: The requirement that f be nonnegative on $[a, b]$ means that no portion of its graph on the interval is below the x -axis. By using rectangles, we can see that there are three different ways of approximating the area for A . This method is commonly known as Riemann Sums.

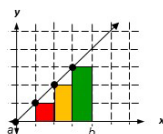
EX #2: Consider a 30-gallon aquarium is being filled with water at a rate modeled by $r(t)$ minutes; and, $r(t)$ is gallons per minute. Find the total amount of water added to the aquarium by using five rectangles that are 1-minute wide.



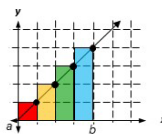
We could achieve a better approximation. In fact, if we use an infinite number of rectangles we would arrive at the definite integral and its value.

Three Types of Riemann Sums

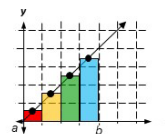
Left-Endpoint (Lower sum)
Inscribed Rectangles



Right-Endpoint (Upper Sum)
Circumscribed Rectangles



Mid-Point (Average)



We can define the area A under the graph on the interval to be as follows.

DEFINITE INTEGRAL DEFINITION

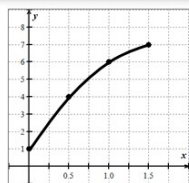
Let f be continuous on $[a, b]$ and $f(x) \geq 0$ for all x in the interval. Let $[a, b]$ be partitioned into n subintervals of equal length $\Delta x = \frac{b-a}{n}$. Then, the definite integral of f over $[a, b]$ is given by:

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x = \int_a^b f(x) dx$$

General Solution Method for Inscribed or Circumscribed Rectangles

1. Draw a rough graph of the function over the interval.
2. Use formula $\Delta x = \frac{b-a}{n}$ to determine each subinterval.
3. Compute x -coordinates of rectangles at either endpoint.
4. Compute the areas of each rectangle (inscribed or circumscribed).
5. Find the summation of the approximated areas.

EX #3: Find the area A under the graph of $f(x) = \frac{1}{2}x + 1$ on the interval $[0, 2]$.



EX #4: For the function shown at right, draw rectangles that represent the Left-Hand Sum shaded in blue.

- A. Calculate the left-hand sum. Let $\Delta x = 2$.

- B. Next, draw the rectangles that represent the Right-Hand Sum. Shade them in red and calculate the area under the curve given by RHS.

- C. Which sum over-estimates, which under-estimates?

- D. Repeat steps A - C for the function shown at right. Let $\Delta x = 0.5$.

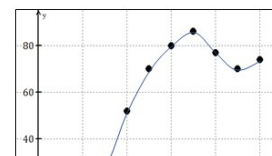
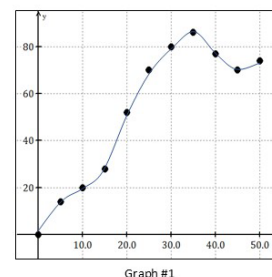
EX #6: The table below represents the velocity, $v(t)$, in feet per second of a car traveling on a straight road over a 50-second time interval. We want to estimate the distance the car traveled over this interval using RHS and LHS along with two different approximation techniques, the midpoint and trapezoid rules. Use the table along with the graph for your computations.

t (sec)	0	5	10	15	20	25	30	35	40	45	50
$v(t)$ (ft/sec)	0	14	20	28	52	70	80	86	77	70	74

- A. Estimate the total distance traveled by the car by using the Left-Hand Sum method, L_5 , with five partitions of equal length. Show your setup and draw the rectangles on graph #1, in blue.

- B. Now use a Right-Hand Sum with 5 partitions, R_5 . Show your setup and draw your rectangles on graph #1, in red.

- C. Next, use the Midpoint Sum with 5 partitions, M_5 . The first rectangle is shown in graph #2. Show your setup.



NOTES

Concave
Down

Left-Hand

Right-Hand

Left-Hand

Right-Hand

6.I

Approximating Areas
with Riemann Sums
Homework

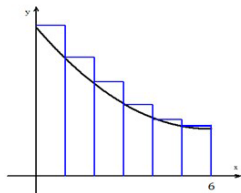
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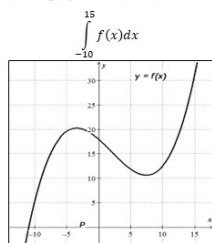
Problems 1 – 10, Complete the following exercises. Show set up for each approximation.

1. A Riemann sum approximation with
- n
- partitions to
- $\int_a^b f(x)dx$
- is shown below.

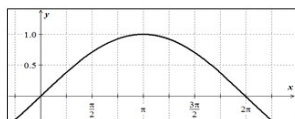
A. Is this a left- or right-hand sum?

B. What are the values of a , b , n , and Δx ?

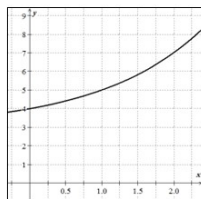
2. Use the graph below to estimate



3. Find the area
- A
- under the graph of
- $f(\theta) = \sin\left(\frac{\theta}{2}\right)$
- on the interval
- $[0, 2\pi]$
- with
- $n=4$
- partitions using Midpoint sums.



4. Find the area
- A
- under the graph of
- $f(x) = 2^x + 3$
- on the interval
- $[0, 2]$
- with
- $n=4$
- partitions using the Trapezoidal sum method.



5. The table below shows the velocity of a small camera drone as it traveled down a golf course fairway for 60 seconds. Estimate the distance traveled by the drone using 6 partitions and the methods below.

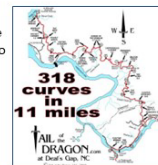
Time (sec.)	0	10	20	30	40	50	60
Velocity (ft/sec.)	0	28	31	33	23	27	15

A. Left-Hand Sums

B. Right-Hand Sums

6. Andy and Bobby were riding their Harley motorcycles on HWY 129 near Robbinsville, NC, heading toward the famous Tail of the Dragon ride. The table below records the time needed to stop the bike before attempting to maneuver the 318 curves. Estimate the total distance needed to stop the motorcycles by using Midpoint rectangles.

Time, in seconds, after braking	0	1	2	3	4	5	6
Velocity, in feet/second	94	76	64	43	27	12	0



7. The table shows the rate at which gas is leaking from a punctured gas tank, in liters/min.

Time (min.)	0	1.3	2.4	3.7	5.5
Rate (liters/min)	5.7	4.3	3.2	2.4	1.6

A right-hand Riemann sum was used to estimate the total number of liters that have leaked from the tank. What is the estimate?

8. Central Florida is the heart of the US Phosphate industry where commercial fertilizer is manufactured. Pollution control devices, known as "wet scrubbers" help capture fluoride gases that are produced to minimize pollution. The EPA requires manufacturers to inspect and report data each month to maintain regulations of toxic waste. Use the midpoint method and six rectangles, in the fictional table, below to estimate the tons of fluoride gases released in the atmosphere over a year.

Time, in months	0	1	2	3	4	5	6	7	8	9	10	11	12
Rate gas escapes in tons/month	3.1	2.4	4.1	3.2	2.8	4.6	5.2	3.9	2.7	2.8	3.3	2.6	2.2

9. While cooking a turkey during the holidays, you decide to run a little data experiment. The table below shows the internal temperature of the turkey when it is placed in the oven at 325°F. Find the average temperature of the turkey over the time interval
- $0 \leq t \leq 30$
- using a left-hand Riemann sum with 5-minute intervals.

Time (min.)	0	5	10	15	20	25	30
Temperature (°F)	42	51	77	81	93	102	117

Practice Sets

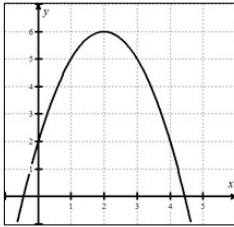
6.1 Approximating Area

Form A

Name _____

Date _____ Period _____

1. Find the area under the curve on $[0,4]$ for $f(x) = 2 + 4x - x^2$ using left-endpoint rectangles and $n = 4$ intervals.



2. Using midpoint approximations find $\int_{-2}^9 h(x) dx$, given the table below:

x	-2	-0.5	1	3	5	7	9
$h(x)$	4	7	1	5	2	6	3

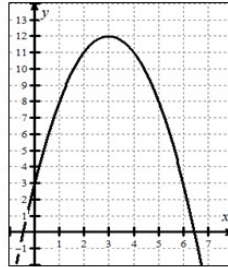
6.1 Approximating Area

Form B

Name _____

Date _____ Period _____

1. Find the area under the curve on $[0,6]$ for $f(x) = 3 + 6x - x^2$ using left-endpoint rectangles and $n = 6$ intervals.



2. Using midpoint approximations find $\int_{-3}^8 f(x) dx$, given the table below:

x	-3
$f(x)$	3

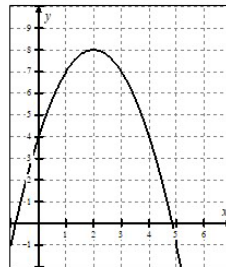
6.1 Approximating Area

Form C

Name _____

Date _____ Period _____

1. Find the area under the curve on $[0, 4]$ for $f(x) = 4 + 4x - x^2$ using right-endpoint rectangles and $n = 4$ intervals.



2. Using midpoint approximations find $\int_{-1}^7 g(x) dx$, given the table below:

x	-1	0	1	3	5	6	7
$f(x)$	0	2	4	5	2	4	0

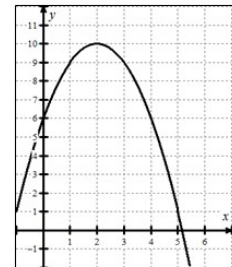
6.1 Approximating Area

Form D

Name _____

Date _____ Period _____

1. Find the area under the curve on $[0, 4]$ for $f(x) = 6 + 4x - x^2$ using right-endpoint rectangles and $n = 4$ intervals.



2. Using midpoint approximations find $\int_{-2}^9 f(x) dx$, given the table below:

x	-3	-1	1	3	5	7	9
$f(x)$	1	2	4	6	8	5	3

Daily Lesson Quiz