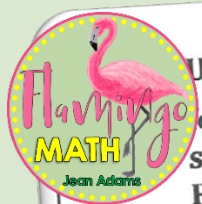


AP[®] Calculus

Basic Derivative Rules



Using the limit definition for the derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ we can derive $f'(x)$ for each of the following trigonometric functions. You will verify the derivative of the sine and cosine functions by using the sum and difference formulas that we learned in Pre-Calculus.

$$\text{Recall: } \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$\text{Recall: } \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0$$

$$\cos(x+h) =$$

$$\cos x \cos h - \sin x \sin h$$

$$\sin(x+h) =$$

$$\sin x \cos h + \cos x \sin h$$

A. Show $\frac{dy}{dx}[\sin x] = \cos x$

$$f'(\sin x) = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

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2.5

Lesson 5: Basic Differentiation Rules

Topic 2.5: Applying the Power Rule

Topic 2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple

1. Derivative of a Constant Function:

The derivative of a constant function is 0.

For any real number, c : $\frac{d}{dx}[c] = 0$



A.) $y = -4$

B.) $s(t) = 45$

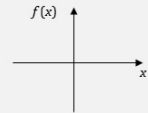
C.) $f(x) = 0$

D.) $y = \frac{k\pi}{2}$

2. The Single Variable Rule:

The derivative of x is 1. $\frac{d}{dx}[x] = 1$

Think graphically about the line $y = x$.



A.) $f(x) = x$

B.) $s(t) = t$

C.) $x(t) = t$

3. The Power Rule:

If n is a rational number, then the function

EX #1: Use the power rule to find the

A.) $y = x^2$

B.)

D.) $g(x) = \frac{1}{x}$

E.)

E. $h(x) = 3x^2 - \frac{4}{x^2} + 2\sqrt{x}$

F. $f(x) = \frac{8x^3 + 4x^2 - 3}{x}$

Topic 2.7: Derivatives of $\cos(x)$, $\sin(x)$, e^x and $\ln(x)$

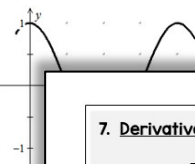
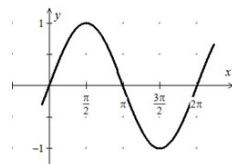
6. Derivatives of Sine, Cosine, and Tangent Functions

EX #4: The derivative of the sine function is the cosine function. Consider where the sine function increases, the cosine function is positive and vice versa.

$\frac{d}{dx}[\sin x] =$

$\frac{d}{dx}[\cos x] =$

$\frac{d}{dx}[\tan x] =$

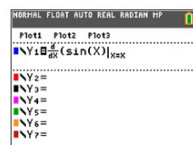


Graphing Derivatives with a Calculator

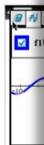
You will enter the function as a function of x , to be evaluated at actual number value.

TI-83+ or TI-84+

Use the numeric derivative operation (MATH > 8) in the graph editor screen.



Use the template



7. Derivatives of the Natural Logarithmic and Exponential Functions:

The derivative of $y = e^x$ is the only function that has itself as its derivative.

If $f(x) = e^x$ then $f'(x) = e^x$

Also,

If $f(x) = \ln x$ then $f'(x) = \frac{1}{x}$, $x > 0$

EX #6: Use the laws of logarithms to find the derivatives of the following:

A.) $f(x) = \ln x^3$

B.) $g(x) = \ln(x^2 \cdot e^x)$

C.) $h(x) = \ln x - \frac{1}{x}$

D.) $y = \ln\left(\frac{2e^x}{3x}\right)$

E.) $f(x) = \ln(4x^3 e^x)$

F.) $g(x) = \pi e^x - \frac{1}{x^2}$

4. The General Power Rule:

The derivative of the term ax^n , where a and n are real numbers, is $(a \cdot n)x^{n-1}$

EX #2: Use the general power rule to find the derivatives of the following, if:

A. $y = 3x^2$

B. $s(t) = -5t^3$

C. $y = 6\sqrt{x}$

D. $g(x) = \frac{2}{x^2}$

E. $f(x) = \frac{4}{\sqrt{x}}$

F. $y = \frac{-9}{\sqrt[3]{x^2}}$

5. The Sum and Difference Rules:

derivative is the sum or difference of the derivatives.

$(f(x) + g'(x))$ and $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Use the Sum and Difference Rules to find the derivatives of the following, if:

B. $y = \frac{2}{5}x^5 + x^4 + 3x^2 - 1$

D. $y = 5x^{3/2} - 6x^{5/3} + \frac{4}{x}$

EX #5: Using the limit definition for the derivative, find the derivative for each of the following trigonometric and cosine functions by using Pre-Calculus.

Recall: $\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$

$\cos(x+h) =$

A. Show $\frac{dy}{dx}[\sin x] = \cos x$

B. Show $\frac{dy}{dx}[\cos x] = -\sin x$

NOTES

$\frac{d}{dx}[\sin x] = f'(\sin x) =$

$\frac{d}{dx}[\cos x] = f'(\cos x) =$

2.5

Basic Differentiation Rules
Homework

Name _____

Date _____ Period _____

Problems 1-14, use the rules of differentiation to find the derivative of the function.

1. $y = 14$

2. $y = x^9 + 3 \tan x$

3. $y = \frac{1}{x^4} - 4 \cos x$

4. $y = \sqrt[3]{x} + 7e^x$

5. $f(x) = 2x - 7 \ln x$

6. $f(x) = 3x^2 + 4x - \ln x^5$

7. $y = -5t^2 + 4t - 3$

8. $g(x) = 9 \sin x - 2x^3$

9. $y = 4e^x - \sqrt{x}$

10. $s(t) = t^3 + 6t^2 - 2 \cos t + 9 \sin t$

11. $h(x) = x(x^3 - 1)$

12. $f(x) = \cos x - \frac{3}{x^2}$

13. $y = (2x - 3)^2$

14. $g(x) = 3e^x + 5 \ln x^3 - 2x$

Problems 15-22, Use algebraic techniques to rewrite each function. Then, find the derivative. Answers should have single term denominators when possible.

15. $f(x) = 2 \sin x + \frac{1}{x^2}$

16. $y = \frac{5x^3 + 7x^2}{x}$

17. $y = \frac{3e^x + 2e^{2x}}{e^x}$

18. $g(x) = \frac{x^3 + 5x^2 + 2}{x^2}$

19. $f(x) = \sqrt{x} + 9 \sqrt[3]{x}$

20. $g(x) = x^{2/3} + x^{1/3} + 2 \sin x$

21. $h(x) = \frac{x - 6\sqrt{x}}{3x^2}$

22. $y = \frac{x^3 - 8}{x - 2}$

23. Find k so that the function $f(x) = x^2 + kx$ will be tangent to the line $y = 2x - 9$

Problems 24-29, Use trigonometric, exponential, and logarithmic derivative rules to find the derivative. Answers should have no negative exponents.

24. $f(x) = 5 \sin x - \ln x^2$

25. $h(x) = \ln(2x^3) - \ln(\sqrt{5x^3})$

26. $y = \ln(\pi x^3) - \frac{2}{\sqrt{x^2}}$

27. $g(x) = \ln\left(\frac{\sqrt{x}}{x^2}\right)$

28. $f(x) = 4 \tan x + 2 \cos x - \ln x^2 - 3e^x$

29. $y = 5e^2 - 2e^x + \sin x - \cos \pi$

Problems 32-33, Find the points on the graph of $f(x)$ where the tangent line is horizontal.

32. $f(x) = 12x - x^3$

33. $g(x) = 4x^{-2} + 8x + 1$

Problems 34-35, Find an equation of the tangent line to the graph at $x = a$.

34. $f(x) = 6x - 16\sqrt{x}$; $x = 4$

35. $g(x) = \sin x + \cos x$; $x = \frac{3\pi}{4}$

Problems 36-37, Solve.

36. Find the values of a and b for $h(x) = x^2 + ax + b$ so $h(1) = 0$ and $h'(1) = 4$.

Practice Sets

2.5 Basic Differentiation Rules Form A

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = 2x^3 + 3x^2 - 4$

2. $g(x) = 3 \sin(x) - 2x^3$

3. $y = \frac{5x-3}{\sqrt{x}}$

4. $f(x) = 5 \tan(x) - \ln x^3$

5. $y = \ln(x^2 \cdot e^x)$

2.5 Basic Differentiation Rules Form B

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = 4x^3 - 5x^2 + 2$

2. $g(x) = x^3 - 3 \cos(x)$

3. $y = \frac{4x-9}{\sqrt{x}}$

4. $f(x) = \ln(x^2) - 3 \cos(x)$

5. $y = \ln(x^2 \cdot e^x)$

2.5 Basic Differentiation Rules Form C

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = 6x^3 - 3x^2 + 5x$

2. $g(x) = 5 \cos(x) - 4x^3$

3. $y = \frac{4x-5}{\sqrt{x}}$

4. $f(x) = \ln(x^4) - 3 \cos(x)$

2.5 Basic Differentiation Rules Form D

Name _____

Date _____ Period _____

Find the derivative. Simplify completely.

1. $f(x) = 4x^3 + 5x^2 + 6x$

2. $g(x) = 6x^3 - 4 \sin(x)$

3. $y = \frac{6x-7}{\sqrt{x}}$

4. $f(x) = 8 \cos(x) - \ln x^5$

Daily Lesson Quiz