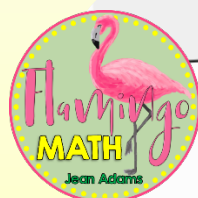


AP[®] Calculus BC

Area and Arc Length in Polar Coordinates

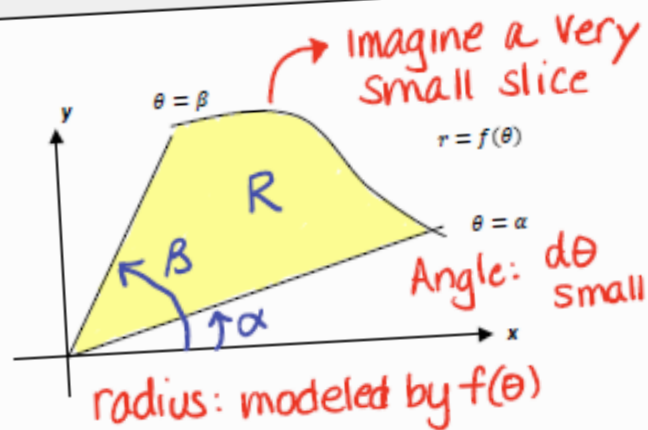


Polar Area Concept

Area of circle (fraction of shaded circle)



$$\text{Sector Area} = \pi r^2 \left(\frac{\theta}{2\pi} \right) = \frac{1}{2} \theta r^2$$



Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial line $\theta = \alpha$ and $\theta = \beta$ is given by

$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta$$

Completely Editable

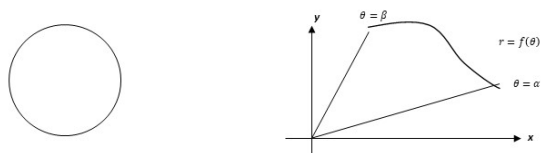
9.6

Lesson 9.6: Area and Arc Length in Polar Coordinates

Topic 9.8: Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve

Let's continue our investigation of polar curves related to areas and arc length. When finding areas in rectangular coordinates, we used rectangles to approximate the region; in polar coordinates, we will use sectors of circles. Another small change will be in our language. In rectangular equations we found the area "under the curve", with polar equations we will find area "enclosed by the region" of the polar curve.

Polar Area Concept



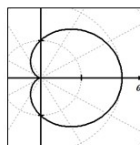
Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial line $\theta = \alpha$ and $\theta = \beta$ is given by

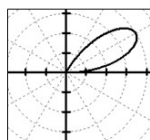
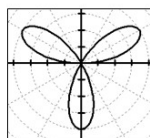
$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta$$

$$= \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

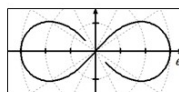
EX #1: Find the area inside the cardioid $r = 1 + \cos \theta$



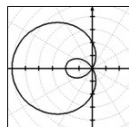
EX #2: Find the area of one petal of $r = 4 \sin(3\theta)$



EX #3: Find the area inside one loop of $r^2 = 9 \cos(2\theta)$.

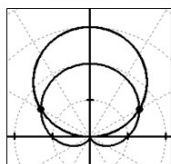


EX #4: Find the area of the inner loop of the graph $r = 2 - 4 \cos(\theta)$

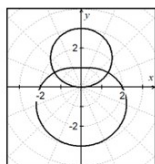


Topic 9.9: Finding the Area of the Region Bounded by Two Polar Curves

EX #5: Find the area of the region that lies inside the circle $r = 3 \sin \theta$ and outside the cardioid $r = 1 + \sin \theta$



EX #6: Find the area of the region that lies inside $r = 3 \sin \theta$ and outside $r = 2 - \sin \theta$.



EX #7: Find the common area of $r = 4 \sin \theta$ and $r = 2$.



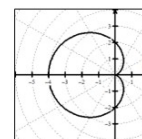
Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$.

The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

EX #8: Find the arc length of the cardioid $r = 2 - 2 \cos \theta$



EX #9: Find the length of the polar curve $r = \sqrt{1 + \cos(2\theta)}$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

NOTES

Using the arc length formula for a parametric curve, we can establish a formula for the arc length of a polar curve.

9.6

Area and Arc Length in Polar Coordinates Homework

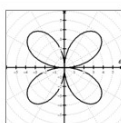
Name _____

Date _____ Period _____

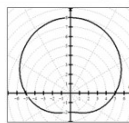
Problems 1 – 6, Find the area of each indicated region.



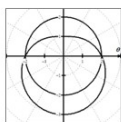
1. One leaf of
- $r = 5 \sin(2\theta)$



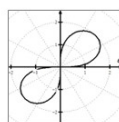
2. Inside
- $r = 5 + 3 \sin(\theta)$



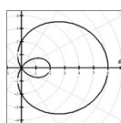
3. Inside
- $r = 2$
- and outside
-
- $r = 2 - \sin(\theta)$



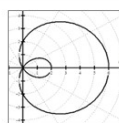
4. Interior of
- $r^2 = 4 \sin(2\theta)$



5. Inside the inner loop of
-
- of
- $r = 2 + 4 \cos(\theta)$

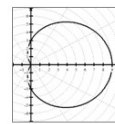
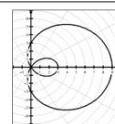


6. Inside the outer loop and
-
- outside the inner loop of
-
- $r = 2 + 4 \cos(\theta)$

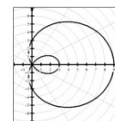


Problems 7 – 11, For each of the following, find the indicated area. Calculators permitted.

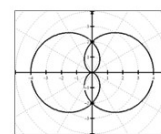
7. The outer loop of
- $r = 3 + 6 \cos(\theta)$



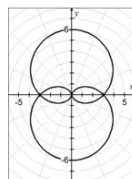
8. Between the loops of
- $r = 3 + 6 \cos(\theta)$



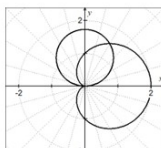
9. The common interior of
- $r = 2 + 2 \cos \theta$
- and
- $r = 2 - 2 \cos \theta$



10. The common interior of
- $r = 3 + 3 \sin \theta$
- and
- $r = 3 - 3 \sin \theta$



11. Inside the circle
- $r = \sqrt{3} \sin \theta$
- and outside the cardioid
- $r = 1 + \cos \theta$



Problems 12 – 15, Find the points of intersection for the graphs of the following. Verify with your calculator.



- 12.
- $r = 2 - \cos \theta$
- ;
- $r = 2 + \sin \theta$

- 13.
- $r = 2 + 4 \sin \theta$
- ;
- $r = 8 \sin \theta$

- 14.
- $r = 3 + 2 \sin \theta$
- ;
- $r = 2 \csc \theta$

- 15.
- $r = 2 \cos \theta$
- ;
- $r = \frac{3}{2} \sec \theta$

Problems 16 – 19, Find the arc length of the polar function on the indicated interval.

- 16.
- $r = \sec \theta$
- ;
- $0 \leq \theta \leq \frac{\pi}{3}$

- 17.
- $r = 4 \sin \theta$
- ;
- $0 \leq \theta \leq \pi$

Practice Sets

9.6 Area and Arc Length In Polar Coordinates

Form A

Name _____

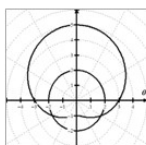
Date _____ Period _____

Given the graphs of the polar curves:

$$r = 2 \text{ and } r = 3 + 2 \sin \theta$$

- Find the polar coordinates where the curves intersect.

- Find the area of the region that lies inside the cardioid and outside the circle.



- Find the length of the cardioid on $[0, 2\pi]$.

9.6 Area and Arc Length In Polar Coordinates

Form B

Name _____

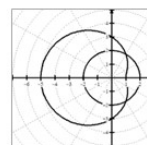
Date _____ Period _____

Given the graphs of the polar curves:

$$r = 2 \text{ and } r = 3 - 2 \cos \theta$$

- Find the polar coordinates where the curves intersect.

- Find the area of the region that lies inside the cardioid and outside the circle.



- Find the length of the cardioid on $[0, 2\pi]$.

Daily Lesson Quiz