

AP[®] Calculus

Area Between Two Curves

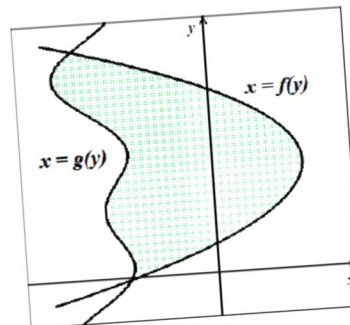
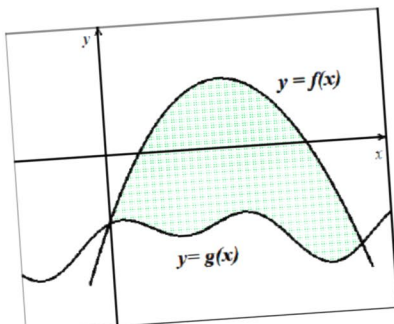


Lesson 4: Area Between Two Curves

8.4: Finding the Area Between Curves Expressed as Functions of x

In this lesson we will explore more applications of integration by investigating the area between two curves and three-dimensional volumes. There are two cases to explore, initially.

Finding the Area Between Curves



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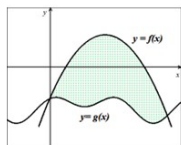
8.4

Lesson 8.4: Area Between Two Curves

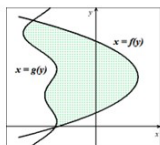
Topic 8.4: Finding the Area Between Curves Expressed as Functions of x

In this lesson we will explore more applications of integration by investigating the area between two curves and three-dimensional volumes. There are two cases to explore, initially.

Finding the Area Between Curves



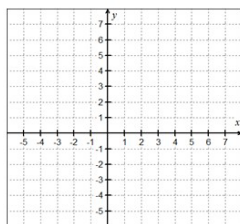
Find the area between $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$ where $f(x) \geq g(x)$.



Find the area between $x = f(y)$ and $x = g(y)$ on the interval $[c, d]$ where $f(y) \geq g(y)$.

EX #1: On the grid at right,

- Graph the function $f(x) = -\frac{1}{2}(x-2)^2 + 6$
- Find the value of $\int_0^5 f(x) dx$
- Shade the region above.
- Graph the function $g(x) = 4 - \frac{1}{2}x$
- Find the value of $\int_0^5 g(x) dx$
- Shade the region below.
- How would you find the area of the region that lies between the two graphs.



General Approach for Finding the Area Between Two Curves

This leads to the general approach we will use for finding the area between two curves.

EX#2: Find the area between the two curves algebraically. Show your work.

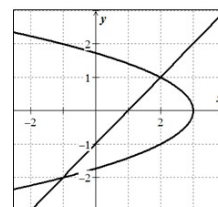
$$A = \int_a^b [f(x) - g(x)] dx$$

Topic 8.5: Finding the Area Between Curves Expressed as Functions of y

EX #3: Horizontal Rectangles (y -Axis Approach)

Find the area of region bounded by graphs of $x = 3 - y^2$ and $x = y + 1$

NOTE: w.r.t. $x \rightarrow$ need two integrals for $[-1, 2]$ and $[2, 3]$
w.r.t. $y \rightarrow$ need only one integral for $[-2, 1]$



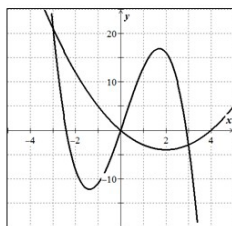
Multiple Points of Intersection

Topic 8.6: Finding the Area Between Curves That Intersect at More Than Two Points

EX #4: Find the Area of the Region between the graphs of

$$f(x) = 14x + x^2 - 2x^3 \text{ and } g(x) = x^2 - 4x$$

A. Find the intersection of the two curves.



B. Write the integrals and calculate the area.

EX #5: Find the area of one of the regions bounded by the sine and cosine curves.

A. Sketch note $f(x) \geq g(x)$

$$f(x) =$$

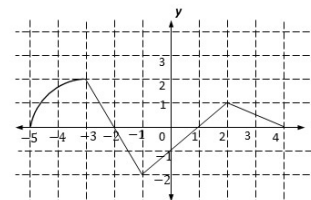


Area as an Accumulation Process - Revised

EX #6: The graph of the function g consists of a quarter-circle and three line segments, shown at right. Let f be the function defined by:

$$f(x) = \int_{-1}^x g(t) dt.$$

- Find $f(-5)$
- Find all the values of x on the open interval $(-5, 4)$ where f is decreasing. Justify your answer.
- Write an equation for the line tangent to the graph of f at $x = -1$.
- Find the minimum value of f on the closed interval $[-5, 4]$. Justify your answer.



NOTES

8.4

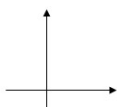
Area Between Two Curves
Homework

Name _____

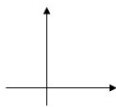
Date _____ Period _____

Problems 1 - 12, Sketch the region bounded by the graphs of the equations and find its area.

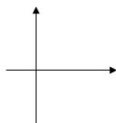
1. $y = x^2$; $y = 4x$



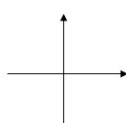
2. $y = x^2 + 1$; $y = 5$



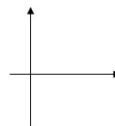
3. $x = y^2$; $x - y = 2$



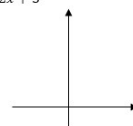
4. $y^2 = 4 + x$; $y^2 + x = 2$



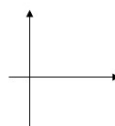
5. $y = (x - 1)^3$; $y = x - 1$



6. $y = x^2 + 2x + 1$; $y = 2x + 5$

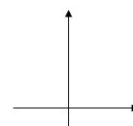


7. $y = x^2 - 4x + 3$; $y = -x^2 + 2x + 3$



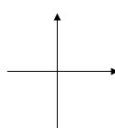
8. [CALCULATOR]

$y = \frac{1}{1+x^2}$; $y = \frac{1}{2}x^2$



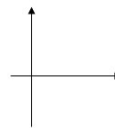
9. [CALCULATOR]

$y = 2 \sin x$; $y = \tan x$; $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

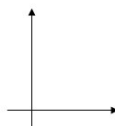


10. [CALCULATOR]

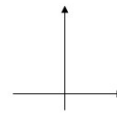
$y = 2 \sin x$; $y = \cos 2x$; $0 \leq x \leq \pi$



11. $y = x^2$; $y = \frac{1}{x^2}$; $y = 9$ Quadrant I



12. $y = 12 - x^2$, $y = -x$, and $y = 4x$ for $y \geq 0$



Practice Sets

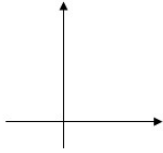
8.4 Area Between Curves Form A

Name _____

Date _____ Period _____

1. Sketch the graphs.

$$f(x) = x^2 \text{ and } g(x) = 3x - x^2$$



2. Locate the interval $[a, b]$ by finding points of intersection.

3. Find the area between the two curves by integration.

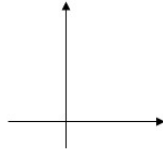
8.4 Area Between Curves Form B

Name _____

Date _____ Period _____

1. Sketch the graphs.

$$f(x) = x^2 - 2x \text{ and } g(x) = 4x - x^2$$



2. Locate the interval $[a, b]$ by finding points of intersection.

3. Find the area between the two curves by integration.

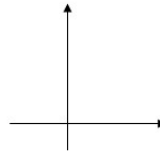
8.4 Area Between Curves Form C

Name _____

Date _____ Period _____

1. Sketch the graphs.

$$f(x) = x^2 + 4x \text{ and } g(x) = 6 - x^2$$



2. Locate the interval $[a, b]$ by finding points of intersection.

3. Find the area between the two curves by integration.

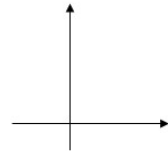
8.4 Area Between Curves Form D

Name _____

Date _____ Period _____

1. Sketch the graphs.

$$f(x) = 3x - x^2 \text{ and } g(x) = x^2 - 2x$$



2. Locate the interval $[a, b]$ by finding points of intersection.

3. Find the area between the two curves by integration.

Daily Lesson Quiz