

Precalculus Honors

Arc Length & Circular Motion

Lesson 3: Arc Length and Circular Motion

From geometry, we found if given a central angle and the length of the radius, we could set up a proportion to find the length of an arc in a circle. **The arc length is a fraction of the total circumference, and the central angle is a fraction of a full revolution.**

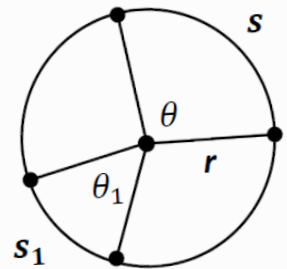
Arc Length Formula

Likewise, from geometry we know that the ratio of the measures of the angles equals the ratio of the corresponding lengths of the arc subtended by these angles.

That is, $\frac{\theta}{\theta_1} = \frac{s}{s_1}$

$$\frac{\theta}{1} = \frac{s}{r} \Rightarrow s = r\theta$$

If we let $\theta_1 = 1$ radian, then $s_1 = r$ from our previous definition of a radian. From here we can find a formula for arc length.



Arc Length Formula

$$s = r\theta$$

Completely Editable

4.3

Lesson 3: Arc Length and Circular Motion

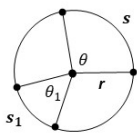
In geometry, we found if given a central angle and the length of the radius, we could set up a proportion to find the length of an arc in a circle. **The arc length is a fraction of the total circumference, and the central angle is a fraction of a full revolution.**

Arc Length Formula

Likewise, from geometry we know that the ratio of the measures of the angles equals the ratio of the corresponding lengths of the arc subtended by these angles.

$$\text{That is, } \frac{\theta}{\theta_1} = \frac{s}{s_1}$$

If we let $\theta_1 = 1$ radian, then $s_1 = r$ from our previous definition of a radian. From here we can find a formula for arc length.



Arc Length Formula

$$s = r\theta$$

(where θ is in radians, never degrees!)

EX #1: Given a central angle of 60° and a radius of 8 cm, find the length of the intercepted arc.



EX #2: If a 100° arc of a circle has a length of 9 inches, what is the radius of the circle?

EX #3: Find the angle in radians, then convert to degrees.

Circular Motion

Linear Velocity (speed)

Have you ever thought about math while out for a bicycle ride? Probably not! You know how to find your average speed by calculating distance traveled divided by the time from long ago (rate = distance/time or $R = D/T$). In trigonometry, we will call this **linear (speed) velocity** and rename our variables using v for velocity (rate) and s (arc length) for distance.

Linear Velocity Formula

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v(\Delta t)$$

EX #4: Dimensional Analysis

If you travel in a car at 65mph, how far did you go in 10 seconds?

Angular Velocity (speed)

$$\frac{\Delta \theta}{\Delta t} \Rightarrow \Delta \theta = \omega(\Delta t)$$

while we ride our bicycle. Think about how fast the wheel is spinning. This is the rate that measures changes in the wheel's angular position.

Angular Velocity Formula

If a wheel rotates 8 radians in 8 seconds, how fast is the angle spinning per second? (in radians per second.)

Units and Converting Between Linear and Angular

Velocity Summary

Linear velocity has dimensions of length per unit of time. Examples, such as, **feet per second, miles per hour, meters per minute.** (The numerator units are measures of length.)

Angular velocity has dimensions of angle measure (radians) per unit of time. Examples, such as **revolutions per minute, radians per second, degrees per hour.** (The numerator units are angle measurements.)

Converting Units of Angular Velocity

EX #6: Convert 60 rpm (revolutions per minute)

A. to radians per minute

B. to degrees per second

Important Relationships between Linear and Angular Velocity

We can use the previous formulas to find a relationship between linear velocity (v) and angular velocity (ω).

Linear Velocity:

$$v = \frac{\Delta s}{\Delta t} \quad \text{Linear velocity}$$
$$v = \frac{r\Delta \theta}{\Delta t} \quad \text{Arc formula}$$
$$v = r \left(\frac{\Delta \theta}{\Delta t} \right) \quad \text{Factoring}$$
$$v = r\omega \quad \text{Angular velocity}$$

Linear Velocity = $r \cdot \omega$

$$v = r \cdot \omega$$

Free-Response Question

EX #9: Some Earthly Discoveries

A: Compute the length in feet of an angle measure of 1 minute on the Earth, given that the radius of the Earth is 3960 miles.

B: What is the length of an angle of one second on the Earth?

Area and Arc Length

Area of Sector

$$A = \frac{1}{2}r^2\theta$$

(where θ is in radians)

EX #7: Find the area of the sector of a circle of radius 5 inches and central angle 60° . Round your answer to two decimal places.

Finding Distance Between Two Cities

EX #8: Memphis, Tennessee is located at $35^\circ 9'$ north latitude. St. Louis, Missouri is located at $29^\circ 57'$ north latitude. Assuming that the two cities lie along the same longitude, find the distance between them.

NOTES

4.3

Arc Length and Circular Motion Homework

Name _____

Date _____ Period _____

Problems 1 – 4, Use the arc length formula $s = r\theta$ to find the missing quantity. Round answers to three decimal places.

1. $r = 5$ miles, $s = 3$ miles, $\theta = ?$

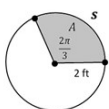
2. $s = 6$ feet $\theta = \frac{1}{2}$ radian, $r = ?$

3. $r = 3$ inches, $\theta = 120^\circ$ $s = ?$

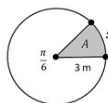
4. $s = 2$ m, $\theta = \frac{1}{3}$ radian, $r = ?$

Problems 5 – 6, Find the length s and area A . Round answers to three decimal places.

5.



6.



Problems 7 – 13, Solve.

7. A strawberry farmer needs to water a strawberry patch of 1500 square yards that is in the shape of a sector of a circle with a radius of 40 yards. Through what angle should the sprinkler rotate?

8. The minute hand of a clock is 6 inches long. How far does the tip of the minute hand move in 15 minutes?

9. A water mill grinds corn to make cornmeal. The water wheel has a radius of 18 feet. The wheel is rotating at 8 revolutions per minute. Find the linear speed, in feet per minute, of the water.

10. A car's wheel with a radius of 1.5 feet is spinning at a rate of 20 revolutions per minute. How fast is the car traveling?

11. Jacksonville, Florida is due south of Charleston, West Virginia. Find the distance between the two cities if Charleston is located at $38^\circ 21'$ North latitude and Jacksonville is $30^\circ 20'$ North latitude. Assume that the radius of the Earth is 3960 miles.

12. It takes twelve identical pieces to form a circular track for a pair of toy electric race cars. If the inside arc of each piece is 3.2 inches shorter than the outside arc, what is the width of the track?

13. A car that has a wheel radius of 1.5 feet travels 30 miles in 45 minutes. How fast are the wheels

Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 4.3 - Arc Length and Circular Motion Warm-Up # I	
1. In a circle with $r = 14$ in, $\theta = 30^\circ$, find the length of arc s . Round to three decimal places.	2. Find the area of the sector whose radius is 5 ft, and $\theta = 4$ radians.
3. The radius of each wheel is 12 inches. If the wheels are turning at the rate of 4 revolutions per second, how fast is the car moving? Write your answer in miles/hour.	

Name _____ Date _____ Period _____	
Lesson 4.3 - Arc Length and Circular Motion	
1. In a circle with $r = 12$ in, $\theta = 45^\circ$, find the length of arc s . Round to three decimal places.	2. Find the area of the sector whose radius is 6 feet and $\theta = 0.75$ radians.
3. The radius of each wheel is 14 inches. If you are traveling at 20 mph, how fast are the wheels turning?	

4.3 Arc Length and Circular Motion Form A
Name _____
Date _____ Period _____
1. In a circle with $r = 10$ ft., $\theta = 60^\circ$, find the length of the arc s . Round answer to three decimal places.
2. Find the area of the sector whose radius is 6 feet and $\theta = 2$ radians.
3. The radius of each wheel of a car is 18 inches. If the wheels are turning at the rate of 3 revolutions per second, how fast is the car moving? Write your answer in inches per second and in miles per hour.

4.3 Arc Length and Circular Motion Form B
Name _____
Date _____ Period _____
1. In a circle with $r = 16$ in., $\theta = 75^\circ$, find the length of the arc s . Round answer to three decimal places.
2. Find the area of the sector whose radius is 10 meters and $\theta = 0.5$ radian.
3. The radius of each wheel of a bicycle is 13 inches. If you are traveling at 25 miles per hour on the bicycle, through how many revolutions per minute are the wheels turning?

Daily Lesson Quiz

Grade: _____

Grade: _____
