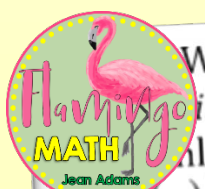


AP[®] Calculus

Antiderivatives and

Indefinite Integrals

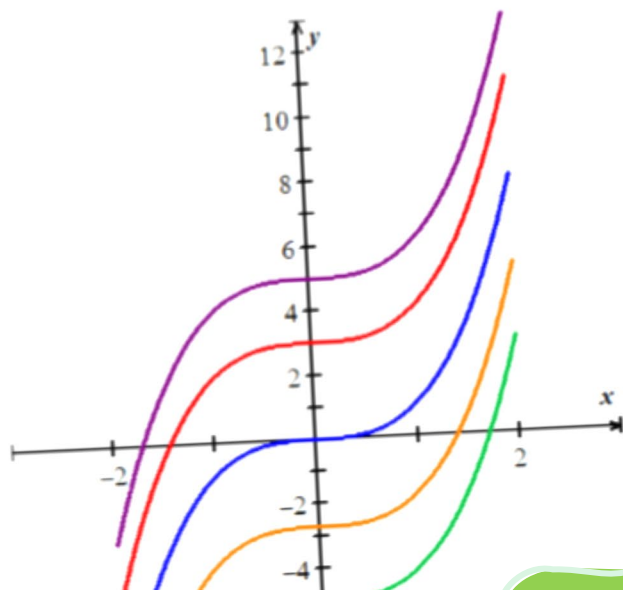


We know that if a function f has an anti-derivative, $F(x)$, then there is a family of functions which share the same anti-derivatives; and, they will differ only by a constant, C . State three functions F_1, F_2, F_3 whose derivatives are $f(x) = 3x^2$.

$F_1(x)$

$F_2(x)$

$F_3(x)$



B. Explain the meaning of “+C” on your functions above.

Completely Editable

6.5

Lesson 6.5: Antiderivatives and Indefinite Integrals Basic Rules and Notation

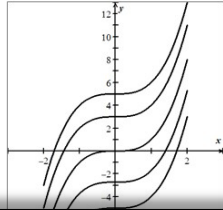
Topic 6.8: Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation

VOCABULARY AND DEFINITIONS:

1. A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f . The process of finding an antiderivative is **antidifferentiation**.
2. The family of all antiderivatives of a function $f(x)$ is the **indefinite integral** of f with respect to x and is denoted by $\int f(x) dx$.
3. If F is any function such that $F'(x) = f(x)$, then $\int f(x) dx = F(x) + C$ is called the **general solution** and C is called the **constant of integration** (an arbitrary constant).

EX #1: We know that if a function f has an anti-derivative, $F(x)$, then there is a "family of functions" which share the same derivatives; and their antiderivatives will differ only by a constant, C .

- A. State three functions F_1, F_2, F_3 whose derivatives are $f(x) = 3x^2$.



- B. Explain the meaning of "+C" on your function.

EX #2: Applying Basic Rules

Given each derivative, below, find the general antiderivative.

A. $g'(x) = 2x$

B. $f'(t) = t^2$

D. $\frac{df}{d\theta} = \cos \theta$

E. $F' = \frac{2}{3}x^{\frac{3}{2}}$

The expression $\int f(x) dx$ is read as the **antiderivative of f with respect to x** . So, the differential dx serves to identify x as the variable of integration. The term "**indefinite integral**" is a synonym for **antiderivative**.

Notation for antiderivatives: $\frac{dy}{dx} = f(x)$
 $\phi y = f(x) dx$

The operation of finding all solutions of this equation is called antidifferentiation or indefinite integration denoted by sign.

A GENERAL ANTI-DERIVATIVE is denoted by:

$$y = \int f(x) dx = F(x) + C$$

EX #3: When integrating, think before you work! Practice will help you "discover" many tricks and techniques that make the integration rules "fit your problems".

**TRICK #1
REWRITE**

$$\int 5e^t + \frac{1}{\cos^2 t} dt$$

$$\int (3x-1)^2 dx$$

EX #4: Match each function with its antiderivative, where C is a constant.

Function	Antiderivative
☐ $f(x) = 6x^2 + 6x - 5$	A. $F(x) = x^3 - 6x^2 + 4x + C$
☐ $f(x) = 3x^2 - 12x + 4$	B. $F(x) = 2x^3 - 4x^2 - 2x + C$
☐ $f(x) = 4x^3 + 6x^2 - 2x$	C. $F(x) = 2x^3 + 3x^2 - 5x + C$
☐ $f(x) = 6x^2 - 8x - 2$	D. $F(x) = x^4 + 2x^3 - x^2 + C$

#4B: For $x > 3$, match the integral to its antiderivative.

Integral	Antiderivative
☐ $\int \frac{1}{x} dx$	E. $F(x) = \ln(2x - 6) + C$
☐ $\int \frac{1}{x-3} dx$	F. $F(x) = \ln x-3 + C$
☐ $\int \frac{2x}{x^2-2} dx$	G. $F(x) = \ln x^2-2 + C$
☐ $\int \frac{3x^2}{x^3-3} dx$	H. $F(x) = \ln x^3-3 + C$

#4C: Match the antiderivative to its integral.

Antiderivative	
☐ $-6 \cos(3x + 4) + C$	J. $\int -6 \sin(3x + 4) dx$
☐ $6 \cos(3x + 1) + C$	K. $\int 6 \sin(3x + 1) dx$
☐ $6 \sin(3x + 4) + C$	L. $\int -6 \cos(3x + 4) dx$
☐ $-6 \sin(3x + 1) + C$	M. $\int 6 \cos(3x + 1) dx$

SOLVING DIFFERENTIAL EQUATIONS

In Example 1, we explored a simple differential equation (DEQ), $\frac{dy}{dx} = 3x^2$.

So, a differential equation is just a statement about the derivative of y (with respect to x). We found a family of functions that had the same derivative. This antiderivative is the **general solution**. Now, our goal will be to solve for y with a **specific solution** for the constant of integration, $+C$, based on an **initial condition** that can be used to find C .

EX #5: Solve the differential equations subject to given conditions.

A. $f'(x) = 2x - 1$, $f(2) = 5$

B. $f'(x) = 2 \cos x$, $f\left(\frac{\pi}{2}\right) = -3$

C. $f'(x) = x^2 + x - 7$, $f(3) = 1$

D. $f'(x) = 3x^2 - 4x + 2$, $f(-1) = -4$

NOTES

dx

$$\frac{d}{dx} [\ln x] = \frac{1}{x} \quad x > 0$$

$\int \frac{1}{x} dx = \ln|x| + C$

$$\int \frac{1}{x} dx = \ln|x| + C$$

6.5

Antiderivatives & Indefinite Integrals Homework

Name _____

Date _____ Period _____

Problems 1 – 14, Find the indefinite integral and check the result by differentiation.

1. $\int (x^3 - 2) dx$

2. $\int (x^{3/2} + 4x + 1) dx$

3. $\int (\sqrt[3]{x^2} - 1) dx$

4. $\int \frac{1}{x^4} dx$

5. $\int \frac{x-3}{\sqrt{x}} dx$

6. $\int (y+3)y^2 dy$

7. $\int 5 dt$

8. $\int (3 \cos x - 2 \sin x) dx$

9. $\int (3 + \sec x \tan x) dx$

10. $\int (\sqrt{x^2} - 2) dx$

11. $\int 5 \sin x - 3e^x dx$

12. $\int (6x^2 + 3e^x) dx$

13. $\int (\tan^2 \theta + 1) d\theta$

14. $\int (\cos x + 5^x) dx$

Problems 15 – 20, Solve the following differential equations to find the particular solution.

15. $f'(x) = \frac{1}{\sqrt{x}}; f(0) = 3$

16. $f'(x) = 6x^2 + 4x; f(1) = -1$

17. $f''(x) = 4; f'(3) = 7; f(3) = -1$

18. $s''(t) = -32; s'(0) = 58; s(0) = 92$

19. $f''(x) = \sin x; f'(0) = -1; f(0) = -4$

20. $g''(x) = x^{-3/2}; g'(4) = 2; g(4) = 6$

Problems 21 – 28, Challenge! Integrate each of the following, by guess and check. Check by differentiating.

21. $\int \cos(3 - 5x) dx$

22. $\int \frac{1}{\sqrt{7-x}} dx$

23. $\int \sin(\theta + 3) [\cos(\theta + 3)]^5 d\theta$

24. $\int (3t - 2)^{100} dt$

25. $\int \frac{e^t + 1}{e^t + t} dt$

26. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

Practice Sets

6.5 Indefinite Integrals**Form A**

Name _____

Date _____ Period _____

1. $\int (6t^2 - 4t + 3) dt$

2. $\int x^5 dx$

3. $\int (10w\sqrt{w}) dw$

4. $\int \frac{1}{\sin^2 \theta} d\theta$

5. $\int (3^x - \sin x) dx$

6.5 Indefinite Integrals**Form B**

Name _____

Date _____ Period _____

1. $\int (9t^2 - 3t + 2) dt$

2. $\int x^6 dx$

3. $\int \sqrt{w} (2w - 3) dw$

4. $\int \frac{\sec \theta}{\cos \theta} d\theta$

5. $\int$

6.5 Indefinite Integrals**Form C**

Name _____

Date _____ Period _____

1. $\int (5t^2 + 2t - 1) dt$

2. $\int x^8 dx$

3. $\int \left(4\sqrt{w} + \frac{1}{\sqrt{w}} \right) dw$

4. $\int \frac{\sin \theta}{\cos^2 \theta} d\theta$

6.5 Indefinite Integrals**Form D**

Name _____

Date _____ Period _____

1. $\int (4t^2 + 7t - 5) dt$

2. $\int x^7 dx$

3. $\int \frac{6w^2}{\sqrt{w}} dw$

4. $\int \frac{\csc \theta}{\tan \theta} d\theta$

Daily Lesson Quiz