

AP[®] Calculus

Another Look at Particle Motion



Over Spring Break our Calculus class went to Kennedy Space Center. I noticed Myles wore a rubberized suit and he was acting a little sketchy as he left his group. Somehow Myles found a way to the roof of the 526-foot-tall building to test out the “bounce factor” of his newly acquired suit by falling from the rooftop.

- How many seconds elapsed before Myles hit the ground?
- With what velocity (in feet per second) does Myles hit the ground?
- Convert your answer to miles per hour. (5,280 feet = 1 mile)



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Lesson 8.2: Another Look at Particle Motion

Topic 8.2: Connecting Position, Velocity, and Acceleration of Functions Using Integrals

Particle motion problems deal with motion along a straight line. This includes horizontal and vertical motion of a falling object. It's important to note that the motion meant the particle was moving along an axis, not along the graph of the function. Now, it is possible for us to work in two directions. For example, if an object is traveling in a straight line, then its velocity can be expressed as the integral of acceleration. Now we will focus on gaining a better understanding for how to model motion problems that will appear on the AP Calculus Exam.

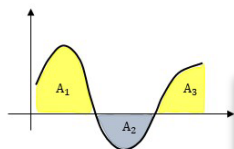
THE NET CHANGE THEOREM:

The definite integral of the rate of change of a quantity $F'(x)$ gives the total change, or **net change**, in that quantity on the interval

$$\int_a^b F'(x) = F(b) - F(a)$$

Three Main Question Stems

When given an object's *velocity* and a question stem is asking about the object's *position*, we need to understand which definite integral is necessary to respond to the prompt accurately.

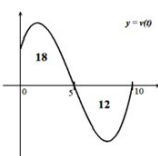


Displacement on $[a, b]$

$$\int_a^b v(t) dt = A_1 - A_2 + A_3$$

EXPLORATION:

Given $s(0) = 4$, find displacement, total distance traveled, and position on $[0, 10]$ where $v(t) =$



Some motion problems will require you to find the actual position of an object or particle at a given point in time. Remember that you will need to use **initial conditions** in order to find an actual position since the definite integral only provides the **change in position of an object**.

EX #3: An object moves in a straight line with velocity $v(t) = \sqrt{3t-2}$ meters per second, where time t is in seconds. At $t = 5$, the object's distance from the starting point was 12 meters in the positive direction.

A. Write an integral expression that will compute the object's position at $t = 10$ seconds.

B. Use a graphing utility to find the actual position of the object at 10 seconds.



EX #4: A particle is moving along the x -axis with velocity $v(t) =$ meters per second after t seconds, with an initial position

A. Determine the displacement of the particle on the time interval

B. What is the total distance traveled by the particle on the time interval

EX #5: Suppose you are driving in traffic belt (this is 45 mph). When you finally get the accelerator, giving the car an acceleration of 10 feet per second per second and $t = 16$ seconds later.

B. How far did you travel during that time interval?

C. What is your average velocity for the time interval?

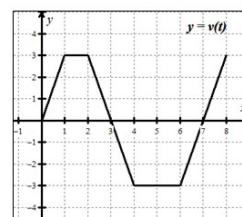
EX #6: You drop your Calculus book from a window 48 feet above the ground.

A. How long will it take your book to reach the ground?



Reviewing Velocity Graphs

EX #1: The graph at right represents the velocity $v(t)$ of a particle that is moving along the x -axis on the interval $0 \leq t \leq 8$, $v(t)$ is measured in inches per second.



A. For what interval(s) of time is the particle moving to the right? Justify your answer.

B. On what interval(s) is the particle slowing down? Justify your answer.

C. When is the particle stopped and changing directions? Justify your answer.

D. On what time interval(s) is the acceleration zero? Justify your answer.

Table Problems

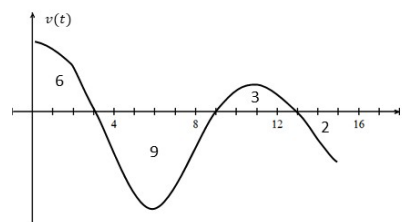
one stop, then decelerates as it comes to another stop. velocity v given in miles per hours, between each stop.

10	20	25	30	40	55	60
6	32	4	36	12	22	0

s travels between every interval (stop) using the trapezoid rule.

aveled in 60 minutes.

EX #7: A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 15$, is given by a differentiable function v whose graph is shown below. At time $t = 0$, the particle is at $x = 4$. The velocity is 0 at $t = 3$, $t = 9$, and $t = 13$. The graph has horizontal tangents at $t = 6$ and $t = 11$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 9]$, $[9, 13]$, and $[13, 15]$ are 6, 9, 3, and 2, respectively.



A. For $0 \leq t \leq 15$, find the time and position of the particle when the particle is farthest to the right. Justify your answer.

B. For how many values of t , where $0 \leq t \leq 15$, is the particle at $x = 4$? Explain your reasoning.

C. During what time intervals, if any, is the acceleration of the particle positive? Justify your answer.

NOTES

8.2

Another Look at Particle Motion Homework

Name _____

Date _____ Period _____

Problems 1 — 4, Find the position $s(t)$ at time t of an object moving on a straight line from the information given about the velocity, acceleration, and position of the object. Find the displacement and distance traveled between time $[0, 1]$.

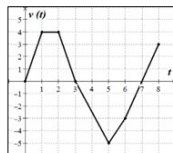
1. $v(t) = 4t^2 - 5t$ and $s(0) = 5$

2. $v(t) = \sin(t)$ and $s(\pi) = -2$

3. $a(t) = -32$, $v(0) = 24$ and $s(0) = 8$

4. $a(t) = 8t$, $s(0) = -6$ and $s(1) = \frac{22}{3}$

5. A particle moves in a straight line with velocity $v(t)$ meters per second, where t is time in seconds. At $t = 4$, the particle's distance from the starting point was 8 meters in the positive direction. Write an expression that will find the particle's displacement between $t = 4$ and $t = 8$.



6. A potato is launched straight up from the ground level and reaches its greatest height at $t = 6$ seconds. Find the initial velocity of the potato and the value of its maximum height above ground level.

7. If a tennis ball is dropped from a height of 60 feet, on planet Newton takes 3 seconds to hit the ground, what is the gravity on the planet?

8. A ball is thrown up from a window, 48 feet above ground level and reaches its maximum height after 3 seconds. How long will it take the ball to reach ground level? What will be the ball's velocity at the instant it reaches ground level?

9. Over Spring Break our Calculus class went to Kennedy Space Center. I noticed Myles wore a rubberized suit and he was acting a little sketchy as he left his group. Somehow Myles found a way to the roof of the 526-foot-tall building to test out the "bounce factor" of his newly acquired suit by falling from the rooftop.



Photo: NASA Photo Archive public domain

A. How many seconds elapsed before Myles hit the ground?

B. With what velocity (in feet per second) does Myles hit the ground?

C. Convert your answer to miles per hour. (5,280 feet = 1 mile)

10. Now, more of the story. Myles wasn't even bruised, thanks to his suit. In fact, he struck the ground and rebounded straight back up into the air with a velocity of 84 feet/sec. How high does Myles fly into the air on the rebound? (Hint: Set $t = 0$ to represent when Myles is on the ground.)

12. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 6t^2 - 36t + 5$, where $s(t)$ is measured in feet and t is measured in seconds.

A. Find the average velocity on the interval $t = 1$ and $t = 8$ seconds. Give correct units.

B. On what interval(s) of time is the particle moving to the left? Justify your answer.

C. Using the correct units, find the velocity and acceleration of the object at $t = 3$ seconds. Based on these values, describe the motion of the object at $t = 3$ seconds. Give a reason for your answer.

D. What is the maximum velocity on the interval from $0 \leq t \leq 8$ seconds. Show your analysis that leads to your conclusion.

Practice Sets

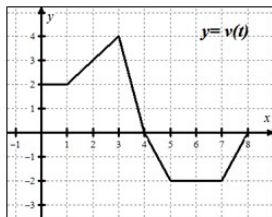
- B. With what velocity will you hit the water if you dive off the platform with an upward velocity of 2 meters per second?

8.2 More on Particle Motion Form A

Name _____

Date _____ Period _____

The velocity of an object moving along a line is given in the graph below for $0 \leq t \leq 8$. At $t = 0$, the position of the object is 10.



1. When is the object at rest?

2. Evaluate $\int_1^6 v(t) dt$. Explain the meaning.

3. Find the position of the object at $t = 5$.

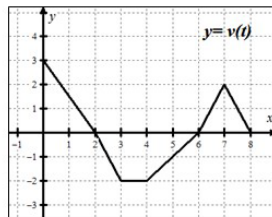
4. What is the total distance traveled by the object on $[0, 8]$?

8.2 More on Particle Motion Form B

Name _____

Date _____ Period _____

The velocity of an object moving along a line is given in the graph below for $0 \leq t \leq 8$. At $t = 0$, the position of the object is 7.



1. When is the object at rest?

2. Evaluate $\int_1^6 v(t) dt$. Explain the meaning.

3. Find the position of the object at $t = 4$.

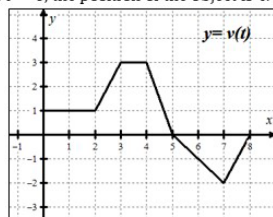
4. What is the total distance traveled by the object on $[0, 8]$?

8.2 More on Particle Motion Form C

Name _____

Date _____ Period _____

The velocity of an object moving along a line is given in the graph below for $0 \leq t \leq 8$. At $t = 0$, the position of the object is 4.



1. When is the object at rest?

2. Evaluate $\int_1^6 v(t) dt$. Explain the meaning.

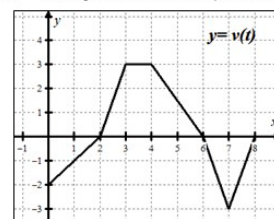
3. Find the position of the object at $t = 6$.

8.2 More on Particle Motion Form D

Name _____

Date _____ Period _____

The velocity of an object moving along a line is given in the graph below for $0 \leq t \leq 8$. At $t = 0$, the position of the object is 9.



1. When is the object at rest?

2. Evaluate $\int_1^6 v(t) dt$. Explain the meaning.

3. Find the position of the object at $t = 4$.

Daily Lesson Quiz