

Precalculus Honors

Angles and Their Measure



Chapter 1: Angles and Their Measure

Engineers in the ancient world discovered ratios within right triangles, as they learned fundamental engineering skills. The secrets and patterns necessary to understanding circles and spheres helped in the development of trigonometry. Our chapter begins the journey into trigonometry with an introduction into angles and their measures.

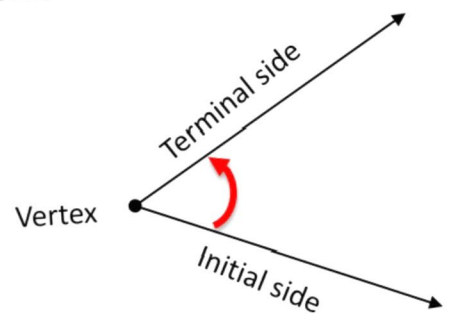
Angle Characteristics

An angle is determined by rotating a **ray** about its **endpoint**.

The vertex is the endpoint of the ray.

The initial side is the starting position.

The terminal side is the position after rotation.



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4.1

Lesson I: Angles and Their Measure

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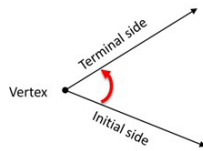
Angle Characteristics

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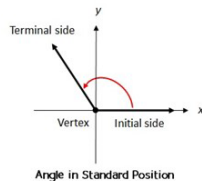


Standard Position Angles

Angles that are in standard position are placed on a coordinate plane, with the vertex at the origin and the initial side on the positive x -axis.

_____ angles are determined by a counterclockwise rotation.

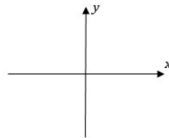
_____ angles are determined by a clockwise rotation.



Coterminal Angles

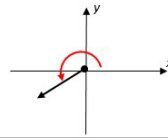
Two angles in standard position that have the same terminal side are called **coterminal** angles. We can find an angle that is coterminal to a given angle by adding or subtracting **one revolution**. So, any given angle has many coterminal angles. For example $\theta = 36^\circ$ is coterminal to all of the following: 396° , 756° , -324° , -684°

EX #1: Draw $\theta = 220^\circ$ and label θ' as the angle that is coterminal.



Reference Angles and Methods to Measure Angles

Reference Angles

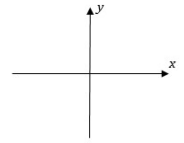
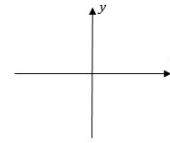


Given an angle in standard position, its reference angle is the **acute** angle formed by the terminal side of the given angle and the x -axis. The angle at left is 210° , the reference angle is _____

EX #2: Draw each angle and state the reference angle.

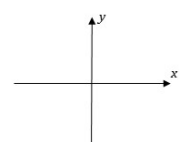
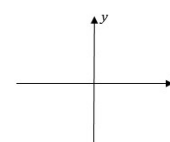
A. 150°

B. 315°



C. -120°

D. 140°



D-M-S Degrees-Minutes-Seconds

We can write decimal degrees in terms of smaller units, called minutes and seconds.

Degrees	Minutes	Seconds
$360^\circ = 1$ revolution	$1^\circ = 60'$	$1' = 60''$ $1^\circ = 3600''$

EX #3: Use the relationships above to manually convert the following.

A. Convert $36^\circ 25' 18''$ to decimal degrees

B. Convert 42.575° to "DMS" form.

Radians and Central Angles

Radians

Another way to measure angles is in _____.

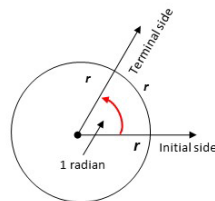
In the late 1800's mathematicians began to see a need for another unit of measure, because they needed angle measurements to "cancel out." This type of measure is needed in calculus to help simplify certain mathematical and physical formulas.

☐ Radians have no units

☐ One complete circular revolution is _____

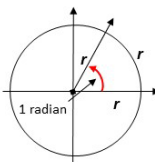
☐ 1 radian is approximately 57.296°

The definition of a radian is based on the concept of a **unit circle**, which is a circle of radius 1 unit, with its center at the origin of a coordinate system. The radian measure of an angle is based on the length of an arc on the unit circle.



Central Angles on the Unit Circle

We use a **central angle** of a unit circle, shown at left, where the vertex is the center of the circle. One radian is the measure of the central angle that subtends (intercepts) an arc equal in length to the radius of the circle. The circumference of any circle is _____, where r is the radius measure. So the circumference of a unit circle where $r = 1$ is _____ units. Therefore, an angle representing one complete revolution of the circle measures _____ radians.



If θ is a central angle in a circle of radius r , and if θ is measured in radians, then the length s of the intercepted arc is given by $s = r\theta$.

Common Radian Measures

Converting Between Units of Angle Measure

Conversion Formulas

Convert Degrees to Radians multiply degrees by:

Convert Radians to Degrees multiple radians by:

EX #4: Convert each angle from radians to degrees or from degrees to radians.

A. $\frac{7\pi}{6}$

D. 135°

B. $-\frac{2\pi}{3}$

E. -320°

C. $-\frac{8\pi}{15}$

F. -110°

Practice With Radians

- How many radians is each angle of an equilateral triangle?
- The minute hand of a clock travels how many radians in 15 minutes?
- The minute hand of a clock travels how many radians in 10 minutes?
- The minute hand of a clock travels how many radians in 5 minutes?

NOTES

4.1

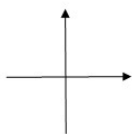
Angles and Their Measure Homework

Name _____

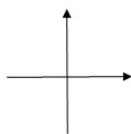
Date _____ Period _____

Problems 1 – 4, Draw each angle, then state the reference angle.

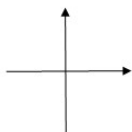
1. 135°



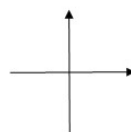
2. 440°



3. $-\frac{\pi}{6}$



4. $\frac{23\pi}{4}$



Problems 5 – 8, Convert each angle to a decimal in degrees. Round your answer to two decimal places.

5. $62^\circ 45' 28''$

6. $105^\circ 18' 7''$

7. $73^\circ 20' 20''$

8. $234^\circ 8' 25''$

Problems 9 – 12, Convert each angle to $D^\circ M' S''$ form. Round

9. 47.35°

10. 18.43°

11. 31.57°

12. 29.99°

Problems 13 – 18, Convert each angle in degrees to radians. Express your answer as a multiple of π .

13. 150°

14. -75°

15. -140°

16. 420°

17. -220°

18. 315°

Problems 19 – 24, Convert each angle in radians to degrees.

19. $\frac{7\pi}{12}$

20. $\frac{9\pi}{2}$

21. $-\frac{5\pi}{6}$

22. 5π

23. $-\frac{5\pi}{3}$

24. $\frac{7\pi}{4}$

Problems 25 – 26, Find the measure of θ in radians and degrees.



Practice Sets

Lesson Warm-Ups

Name _____ Date _____ Period _____	
Lesson 4.1 - Angles and Their Measures Warm-Up # 1	
1. How many degrees is $\frac{\pi}{15}$ radians?	2. Convert 62.475° to DMS.
3. Convert the measure from radian to degree. $\theta = \frac{2\pi}{3}$	4. Given $\theta = \frac{13\pi}{9}$, name the reference angle in radian measure.

Name _____ Date _____	
Lesson 4.1 - Angles and Their Measures	
1. How many degrees is $\frac{2\pi}{5}$ radians?	2. Convert 114.324° to
3. Convert the measure from radian to degree. $\theta = \frac{5\pi}{6}$	4. Given $\theta = \frac{5\pi}{12}$, name

4.1 Angles and Their Measure Form A	4.1 Angles and Their Measure Form B
Name _____	Name _____
Date _____ Period _____	Date _____ Period _____
1. How many degrees in $\frac{\pi}{10}$ radians?	1. How many degrees in $\frac{3\pi}{5}$ radians?
2. Convert 37.245° to DMS.	2. Convert 123.578° to DMS.
3. Convert the measure from radian to degree $\theta = \frac{4\pi}{9}$	3. Convert the measure from radian to degree $\theta = \frac{11\pi}{6}$
4. Given $\theta = \frac{17\pi}{12}$, name the reference angle in radian measure.	4. Given $\theta = \frac{7\pi}{9}$, name the reference angle in radian measure.

Daily Lesson Quiz

Grade: _____	Grade: _____
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