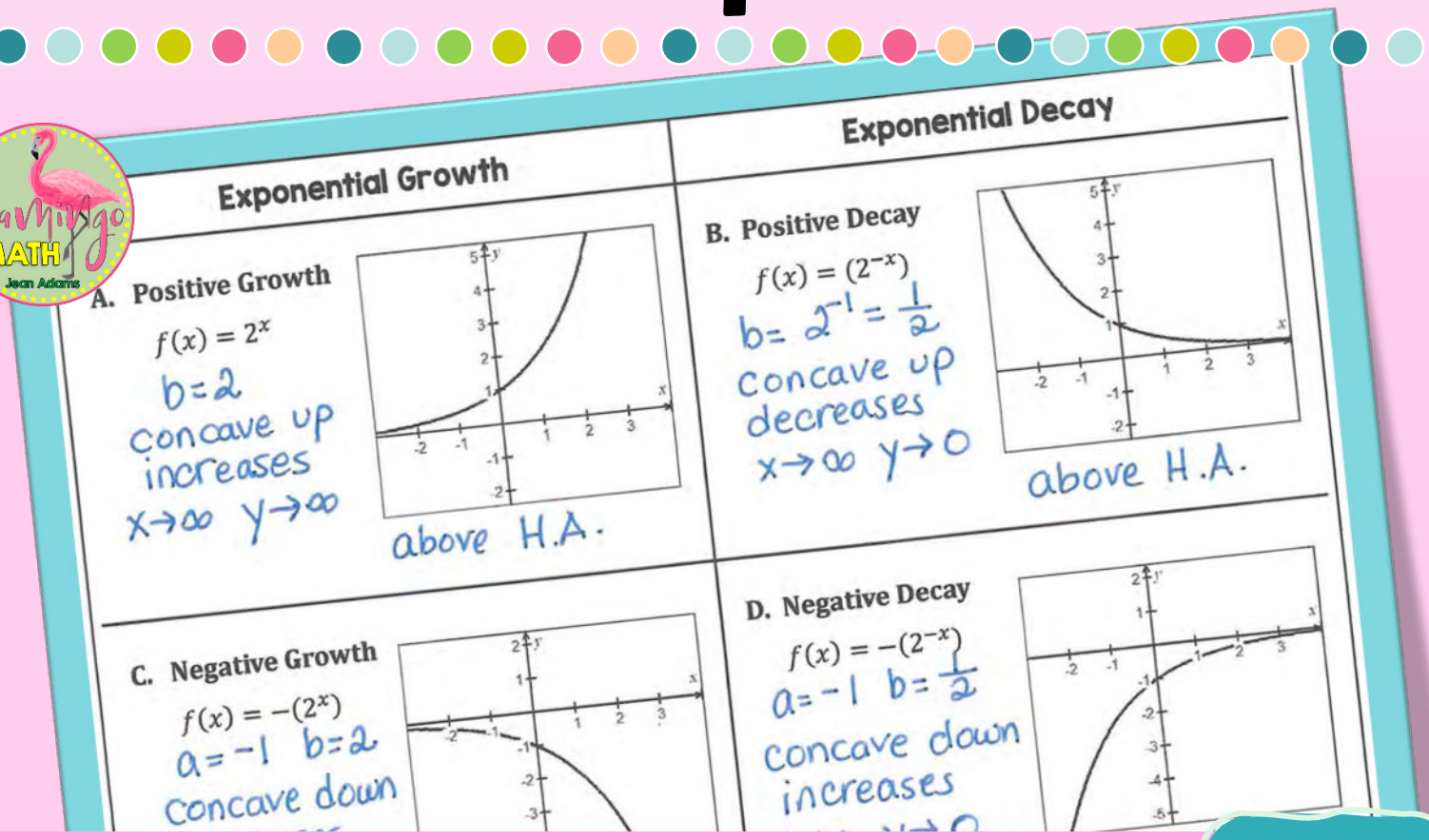
Precalculus Honors

# Analyzing Exponential Graphs




Completely Editable

3.6

## Lesson 6: Analyzing Exponential Graphs

### Growth and decay are not synonymous with increasing and decreasing.

In this unit, we have seen a wide variety of equations and graphs. Moving forward you need to distinguish between the difference between exponential growth and decay models. It is important to consider the behavior of the function in relation to the horizontal asymptote. Notice whether the function is moving away from or toward the horizontal asymptote. We have explored the following formulas that normally represent growth and decay in real-world applications:

| Exponential Growth<br>$k > 0$ | Exponential Decay<br>$k < 0$ |
|---|---|
| $A(t) = A_0 e^{kt}$ | $A(t) = A_0 e^{kt}$ |
| $P(t) = P_0(1+r)^t$ | $P(t) = P_0(1-r)^t$ |
| $y = Ce^{kt}$ | $T(t) = T_m + (T_0 - T_m)e^{kt}$ |

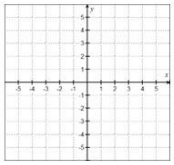
We will need to find the behavior of a graph without the aid of a calculator in this course. By using the rules for transformations of exponential functions, you can find patterns that will be helpful to graphing.

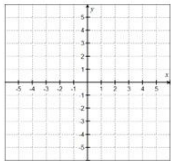
### Analyzing Exponential Functions

**EX #1:** Recall that exponential equations are written in the form $y = ab^{(x-h)} + k$.
In this unit we have seen the following rules without considering the behavior for negative $a$-values.
**Let's explore the effect a negative coefficient on $x$ does to the behavior of a function.**

A. Graph and analyze: $y = -\left(\frac{1}{2}\right)^x$

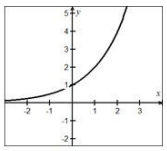
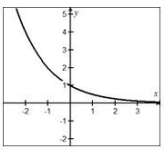
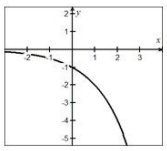
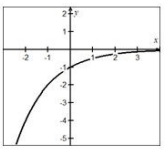
B. Graph and analyze: $y = -2^x$





There are four different versions of exponential function behavior. There can be positive or negative growth and positive or negative decay. Consider the positive growth of money, like investing in a stock or negative growth like the debt of the United States becoming more negative each day. There can also be positive decay, like the half-life of a radioactive element, or negative decay like paying off a loan at half of the loan in the first year, ¼ in the second year, 1/8 in the third year, and so on.

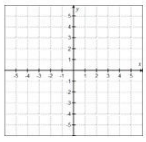
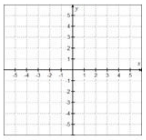
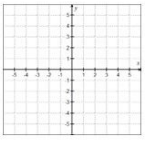
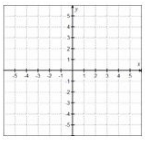
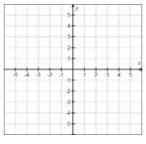
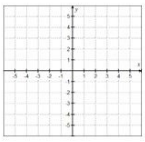
**EX #2:** Analyze the following graphs. Resolve the negative coefficient on the variable prior to stating the base value.

| Exponential Growth | Exponential Decay |
|---|---|
| **A. Positive Growth** $f(x) = 2^x$  | **B. Positive Decay** $f(x) = (2^{-x})$  |
| **C. Negative Growth** $f(x) = -(2^x)$  | **D. Negative Decay** $f(x) = -(2^{-x})$  |

**Complete the table.**

| $y = ab^x$ | |
|---|---|
| $a > 0$ and $b > 1$ | $a > 0$ and $0 < b < 1$ |
| | |
| $a < 0$ and $b > 1$ | $a < 0$ and $0 < b < 1$ |
| | |

**EX #3:** Graph, describe the behavior as increasing or decreasing and state the end-behavior.

| Exponential Growth | Exponential Decay |
|---|---|
| **A.** $f(x) = \left(\frac{3}{2}\right)^{x-1} - 2$  | **D.** $f(x) = (2)^{-x-2} - 3$  |
| **B.** $f(x) = -e^{x-1} + 3$  | **E.** $f(x) = -\left(\frac{1}{4}\right)^{x+1} + 2$  |
| **C.** $f(x) = \left(\frac{1}{2}\right)^{-x}$  | **F.** $f(x) = 3^{-x} - 2$  |

**EX #4:** The graph at right can be modeled by the equation $y = a \cdot b^{-x} + c$. Complete the following.

Growth or Decay Justification:

**EX #5:** Now, use the equations from example #3 to complete the table below:

| Graph of $f(x)$ | Base | Increase or Decrease | $\lim_{x \to -\infty} f(x)$ | $\lim_{x \to \infty} f(x)$ |
|---|---|---|---|---|
| **GROWTH** | $b > 1$ | | | |
| $f(x) = \left(\frac{3}{2}\right)^{x-1} - 2$ | | | | |
| $f(x) = -e^{x-1} + 3$ | | | | |
| $f(x) = \left(\frac{1}{2}\right)^{-x}$ | | | | |
| **DECAY** | $0 < b < 1$ | | | |
| $f(x) = (2)^{-x-2} - 3$ | | | | |
| $f(x) = -\left(\frac{1}{4}\right)^{x+1} + 2$ | | | | |
| $f(x) = 3^{-x} - 2$ | | | | |

**EX #6:** Find the equation of $h(x) = a \cdot b^x + c$, given $h(0) = 3$ $h(-4) = -3$ and as $x \to \infty, h(x) \to 4$.

# NOTES

**3.6** **Analyzing Exponential Graphs Homework**

Name _____

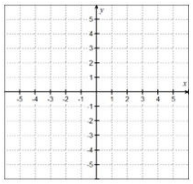
Date _____ Period _____

**Problems 1 – 4**, For each function in the form $y = a \cdot b^{x-h} + k$ Complete the table below.

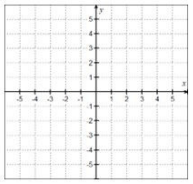
| Function | Value of $a$ | Base | Asymptote | Growth or Decay | Increase or Decrease |
|---|---|---|---|---|---|
| 1. $f(x) = 2^{-x} - 4$ | | | | | |
| 2. $g(x) = -\left(\frac{1}{4}\right)^{-x} + 3$ | | | | | |
| 3. $h(x) = 2\left(\frac{1}{3}\right)^{x} + 1$ | | | | | |
| 4. $k(x) = -(0.5)^{x-1}$ | | | | | |

**Problems 5 – 8**, Use the equations and the table above to sketch the graph of each function.

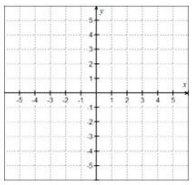
5. $f(x) = 2^{-x} - 4$



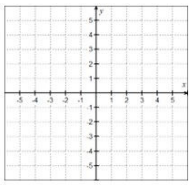
6. $g(x) = -\left(\frac{1}{4}\right)^{-x} + 3$



7. $h(x) = 2\left(\frac{1}{3}\right)^{x} + 1$



8. $k(x) = -(0.5)^{x-1}$



**Problems 9 – 10**, Complete the following for each graph.

9. The graph at right can be modeled by the equation $y = a \cdot b^{x} + c$. Complete the following.

*a:* _____

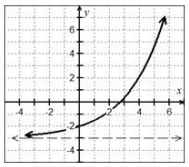
*b:* _____

*c:* _____

*Left end behavior:* _____

*Right end behavior:* _____

Growth or Decay Justification:



10. The graph at right can be modeled by the equation $y = a \cdot b^{-x} + c$. Complete the following.

*a:* _____

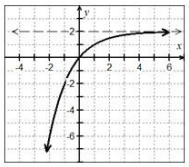
*b:* _____

*c:* _____

*Left end behavior:* _____

*Right end behavior:* _____

Growth or Decay Justification:



**Problems 11 – 12,** Find the equation of the exponential functions. Show work that leads to your answer.

11. For $f(x) = a \cdot b^{x}$ given $f(0) = -5$ and $f(1) = -15$.

12. For $g(x) = a \cdot b^{x} + c$, given $g(0) = 3$ and $g(2) = -21$. Also, as $x \to -\infty, y \to 6$

**Problems 13 – 16,** For the following functions,

(a) Determine whether the graph will display exponential growth or decay. Include whether this behavior is in the positive or negative direction. Give a reason for your conclusion.

(b) State the end behavior of the function; and,

(c) State whether the graph lies above or below the horizontal asymptote.

13. $f(x) = -(0.75)^{x+2} - 4$

14. $g(x) = \left(\frac{2}{3}\right)^{-x} + 3$

15. $h(x) = -3^{x+1} - 2$

**Free Response Question**

17. The table of values for an exponential function can be modeled by $G(x) = a \cdot b^{x} + c$ is given below.

| $x$ | −8 | −6 | −3 | 0 | 1 | 7 | 9 |
|---|---|---|---|---|---|---|---|
| $G(x)$ | −388 | −100 | −16 | −5.5 | −4.75 | −4.011 | −4.002 |

A. State the end behavior for the function.

B. Determine the domain and range of $G(x)$

C. What is the horizontal asymptote? Explain your reasoning.

D. If $a = -\frac{3}{2}$ use the table of values to find an equation for the function $G(x)$. Show your work.

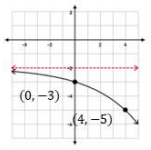
# Practice Sets

# Lesson Warm-Ups

Name _____ Date _____ Period _____

**Lesson 3.6: Analyzing Exponential Graphs: Warm-Up # 1**

**1.** What is the value of $c$ in the equation?




**2.** Find the value of $a$ using $f(0) = -3$.

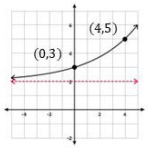
**3.** Determine the value of $b$ to three decimal places.

**4.** Write the equation for $f(x)$.

Name _____ Date _____ Period _____

**Lesson 3.6: Analyzing Exponential Graphs: Warm-Up # 2**

**1.** What is the value of $c$ in the equation?




**2.** Find the value of $a$ using $f(0) = 3$.

**3.** Determine the value of $b$ to three decimal places.

**4.** Write the equation for $f(x)$.

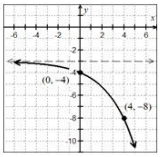
# Daily Lesson Quiz

**3.6 Analyzing Exponential Graphs** **Form A**

Name _____

Date _____ Period _____

**Questions 1 – 2:** The graph of an exponential function of the form $f(x) = a \cdot b^x + c$ is shown below.




1. What is the value of $c$ in the equation?

2. Find the value of $a$ using $f(0) = -4$

3. Determine the value of $b$ to three decimal places.

4. Write the equation for $f(x)$.

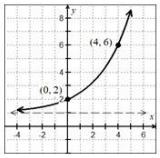
**Grade:**

**3.6 Analyzing Exponential Graphs** **Form B**

Name _____

Date _____ Period _____

**Questions 1 – 2:** The graph of an exponential function of the form $f(x) = a \cdot b^x + c$ is shown below.




1. What is the value of $c$ in the equation?

2. Find the value of $a$ using $f(0) = 2$

3. Determine the value of $b$ to three decimal places.

4. Write the equation for $f(x)$.

**Grade:**