

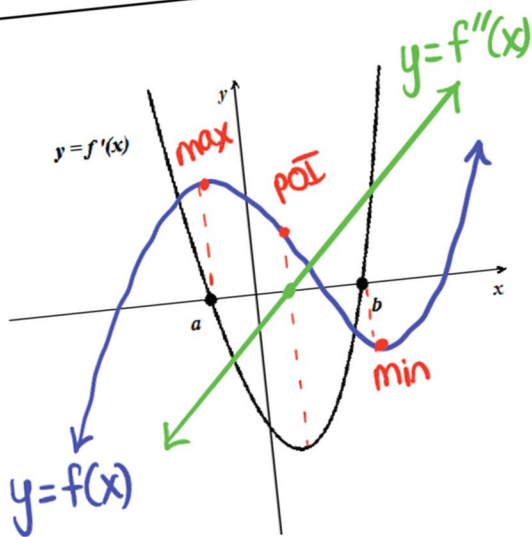
AP[®] Calculus

Analytical Applications of the Derivative

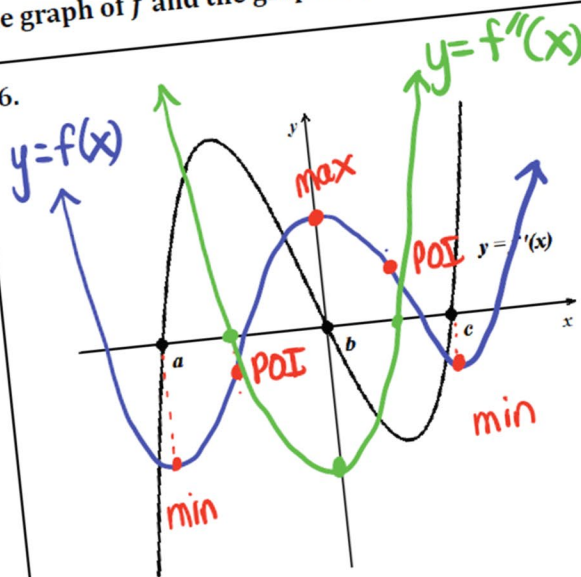


Problems 5 – 6, Use the graph of the f' to sketch the graph of f and the graph of f'' .

5.



6.



of the function, f , given the following information.

Homework

5

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AP CALCULUS CURRICULUM

Scope and Sequence

Unit 5: Analytic Applications of Differentiation

Lesson	Skills & Learning Objectives
[5.1] The Mean Value Theorem	☐ Apply the Mean Value Theorem to describe the behavior of a function over an interval
[5.2] Extrema on an Interval	☐ Use derivatives to find local and global extrema ☐ Use both open interval and closed interval methods for finding extrema. (Candidates Test) ☐ Use and apply the Extreme Value Theorem on intervals
[5.3] Increasing, Decreasing & First Derivative Test	☐ Use the first derivative test to find intervals of increase and decrease, maximum and minimum values
[5.4] Concavity & Second Derivative Test	☐ Use the second derivative test to find points of inflection, intervals where the function is concave up or concave down, and to find maximum and minimum values
[5.5] Sketching Graphs of Functions F , F' and F''	☐ Use the first and second derivative tests to analyze and sketch functions
[5.6] Optimization	☐ Optimize real-world situations by finding values that maximize or minimize constraints

5.1

The Mean Value Theorem
Homework

Name _____

Date _____ Period _____

Problems 1 – 4, Determine whether Rolle's Theorem can be applied to the function on the closed interval. If Rolle's Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

1. $f(x) = x^2 - 3x - 10$ $[-2, 5]$

2. $f(x) = x^{2/3} - 2$ $[-1, 1]$

3. $f(x) = \sin x$ $[0, 2\pi]$

4. $f(x) = x - 2 \ln x$ $[1, 3]$

Problems 5 – 10, Determine whether the Mean Value Theorem can be applied to the function on the closed interval. If the Mean Value Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

5. $f(x) = x^3 - x^2 - 2x$ $[-1, 1]$

6. $f(x) = \sqrt{3 - x}$ $[-6, 3]$

5.2

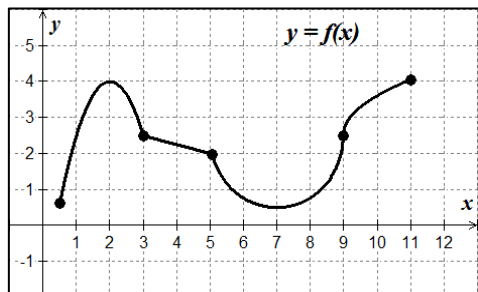
Extrema on an Interval Homework

Name _____

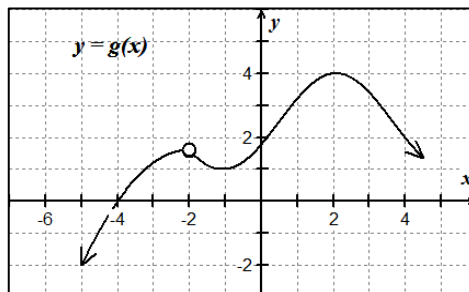
Date _____ Period _____

Problems 1 – 4, use the graph to estimate the absolute extrema and local extrema.

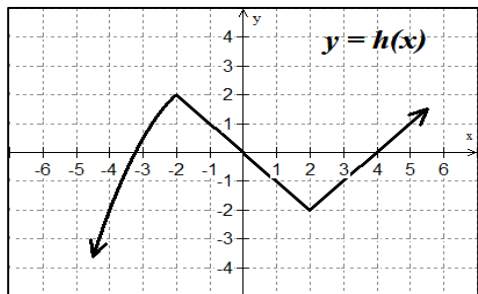
1.



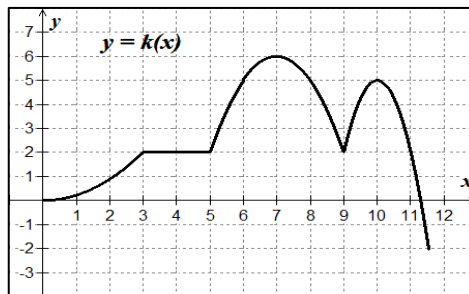
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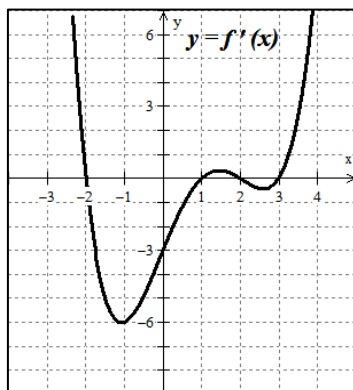


Problems 5 – 10, determine the critical numbers for each function below.

5. $f(x) = \ln(x^2 - 4)$

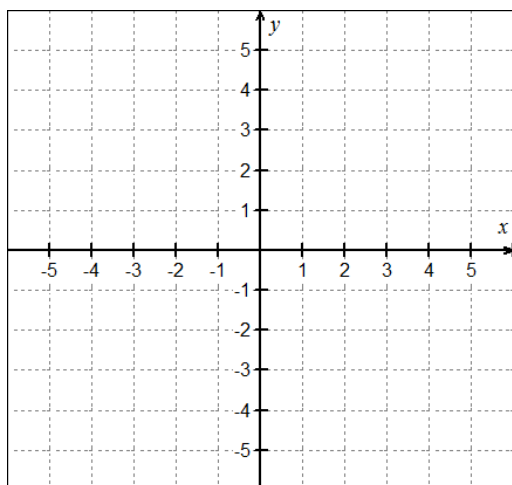
6. $h(x) = \sqrt{x^2 - 9}$

11.

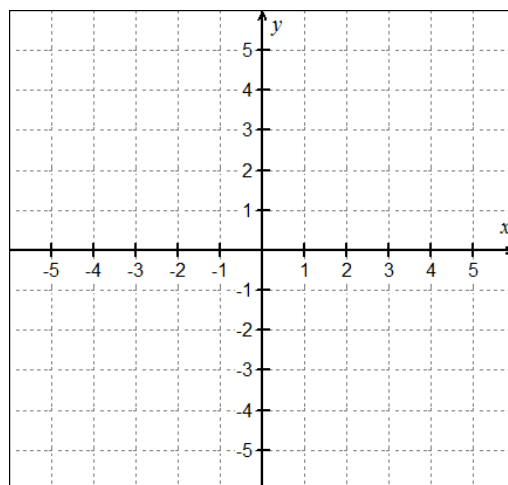


Problems 12 - 13; Sketch the graph of a continuous function f that satisfies all the stated conditions.

- 12.** $f(0) = 1$; $f(2) = 3$; $f'(0) = f'(2) = 0$;
 $f'(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$
 $f'(x) > 0$ on $(0, 2)$



- 13.** $f(1) = f(5) = 0$; $f'(3)$ does not exist
 $f'(x) < 0$ if $x < 3$; $f'(x) > 0$ if $x > 3$
 $f(x)$ is concave down if $x \neq 3$



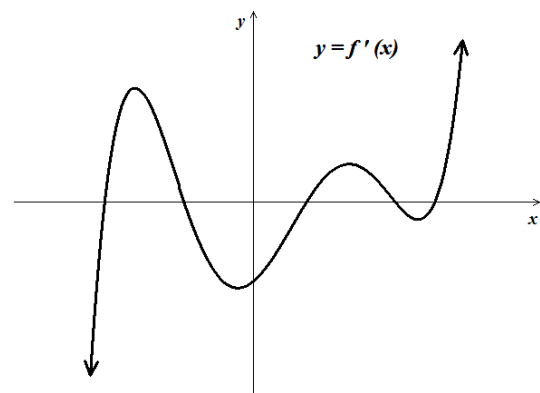
- 14.** Assume f is a differentiable function, where the signs of $f'(x)$ are as follows:

$$f'(x) > 0 \text{ on } (-\infty, -5) \cup (4, \infty) \text{ and } f'(x) < 0 \text{ on } (-5, 4).$$

Let $g(x)$ be a transformation of $f(x)$. Complete the inequality ($>$, $\geq$, $<$, $\leq$) for the value of $x = c$:

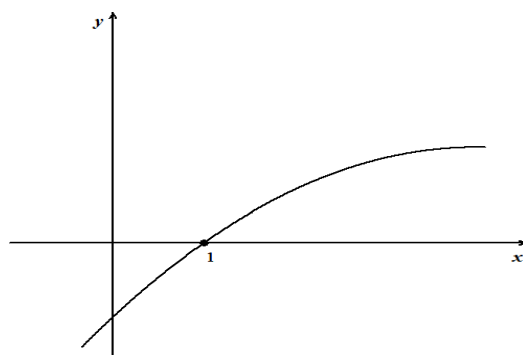
Function	Sign of $g'(c)$
A) $g(x) = -f(x)$	$g'(-8)$ 0
B) $g(x) = f(x - 8)$	$g'(1)$ 0
C) $g(x) = f(2x) - 4$	$g'(-4)$ 0

13. The figure at right, shows the graph of the derivative of a function, f . How many points of inflection does f have in the interval shown. Justify how you know.

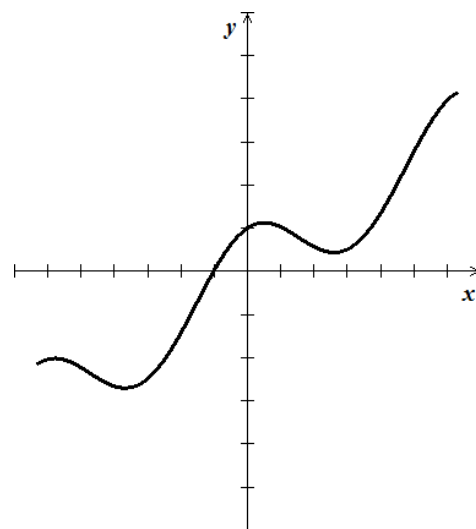


14. The graph of a twice-differentiable function f is shown at right. Which of the following is true?

- A. $f(1) < f'(1) < f''(1)$
- B. $f(1) < f''(1) < f'(1)$
- C. $f'(1) < f(1) < f''(1)$
- D. $f''(1) < f(1) < f'(1)$

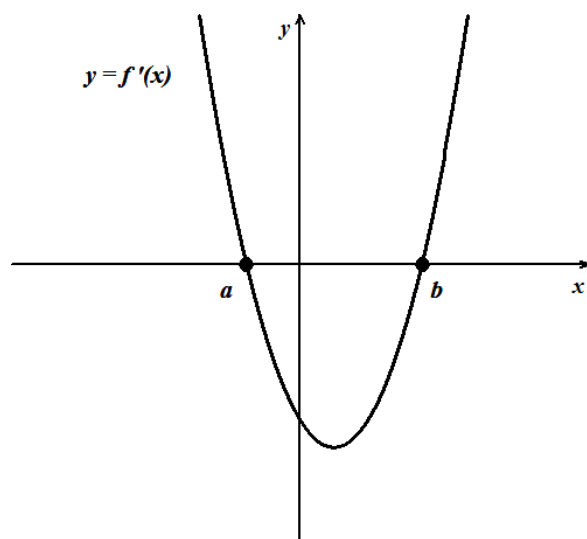


15. The figure at right is a graph of the equation $y = \frac{1}{2}x + \cos x$ on the interval $-2\pi \leq x \leq 2\pi$. Use the second derivative test to find the local extrema of f .

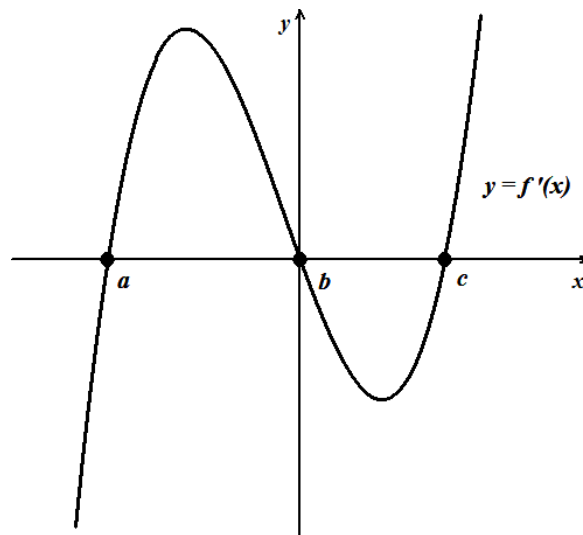


Problems 5 – 6, Use the graph of the f' to sketch the graph of f and the graph of f'' .

5.



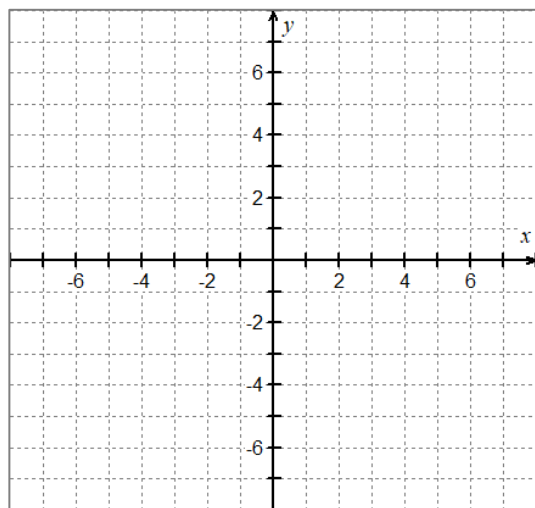
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Problems 7 – 8, Sketch the graph of the function, f , given the following information.

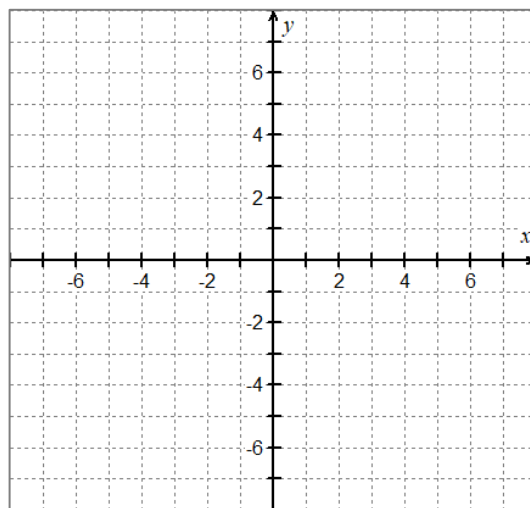
7.

- A. continuous for all real numbers
- B. $f'(x) < 0$ on $(-\infty, -3)$ and $(0, 3)$
- C. $f'(x) > 0$ on $(-3, 0)$ and $(3, \infty)$
- D. $f''(x) > 0$ on $(-\infty, 0)$ and $(0, 6)$
- E. $f''(x) < 0$ on $(6, \infty)$
- F. $f'(0)$ does not exist



8.

- A. continuous for all real numbers
- B. $f'(2)$ does not exist
- C. $\lim_{x \rightarrow -\infty} f(x) = 2$
- D. $f'(x) > 0$ on $(-\infty, -1)$ and $(2, 5)$
- E. $f'(x) < 0$ on $(-1, 2)$ and $(5, \infty)$
- F. $f''(x) > 0$ on $(-\infty, -3)$
- G. $f''(x) < 0$ on $(-3, 2)$ and $(2, \infty)$



Solving Optimization Problems Homework

Name _____

Date _____ Period _____

Problems 1 – 9, solve each question and justify your reasoning.

1. A farmer has 350 feet of fencing to enclose 2 adjacent horse corrals. What dimensions should be used so that the enclosed area will be a maximum?
2. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of the rectangle?
3. The owner of an orange grove estimates that if 24 trees are planted per acre, then each mature tree will yield 600 oranges per year. For each additional tree planted per acre, the number of oranges produced by each tree decrease by 12 per year. How many trees should be planted per acre to obtain the most oranges per year?