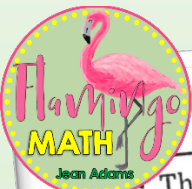


AP[®] Calculus

Accumulation

Functions in Context

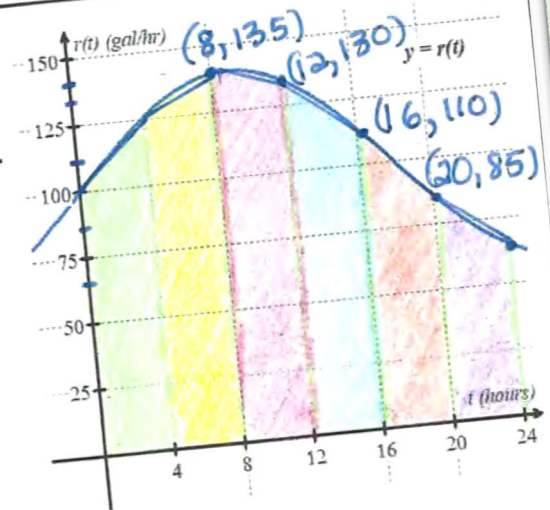


Accumulation from a Graph

The graph at right shows the rate in gallons per hour, at which storm water is being pumped from an overflowing canal. Find the approximate number of gallons pumped from the canal in the 24-hour period.

$$\text{Gallons of Storm Water} = \int_0^{24} r(t) dt$$

$$\approx \frac{1}{2}(4)[100 + 2(125) + 2(135) + 2(130) + 2(110) + 2(85) + 65]$$



Completely Editable

8.3

Lesson 8.3: Accumulation Functions in Applied Contexts

Topic 8.3: Using Accumulation Functions and Definite Integrals in Applied Contexts

The velocity of an object in motion is not the only rate that we can integrate to get an amount. We have seen that accumulation or net change problems present us with a scenario where we are often asked to calculate the value of some quantity accumulated over time. We can solve the problems using definite integrals.

EX #1: You received a 10-week-old basset hound puppy for your birthday and discovered that he is gaining weight at a rate of $e^{0.0125t}$ pounds per week, where t is the puppy's age in weeks. At 10 weeks, the puppy initially weighed 14 pounds. What is the expected weight of your new puppy at the end of week 15?



EX #2: An electric monorail transport is slowing down at a rate of $2t + 25$ feet per second per second, where t is the time in seconds. By how many feet per second does the monorail slow down between $t = 1$ and $t = 5$ seconds?

EX #3: The number of liters of oil in a tank is changing at a rate $r(t) = 0.26t$ liters per minute, where t is the time in minutes. Initially, the tank contained 14 liters of oil. Write an expression that can be used to find the amount of oil in the tank after 10 minutes.

Interpreting Expressions

EX #4: Describe the meaning of each expression in context to the given scenario.

A. The function $w(t)$ gives the number of water bottles produced by a manufacturer by time t (in hours) on any given day. Explain the meaning of the expression:

$$\int_4^8 w'(t) dt$$

B. The amount of oil being drained from your pickup truck is decreasing at a rate of $r(t)$ quarts per minute (where t is the time in minutes). What does the expression below mean?

$$\int_3^4 r(t) dt = -0.5$$

C. Kevin got a gift of \$500 from his grandparents for graduation. He deposited the gift into his college savings account. Kevin's savings grows at a rate of $r(t)$ hundreds of dollars each month (where t is the month of the year). What does the expression below mean?

$$5 + \int_1^{12} r(t) dt = 24$$

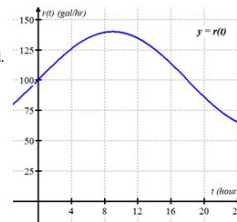
Table Problems

EX #5: A city bus is accelerating from 0 to 18 seconds. The data for the acceleration $a(t)$ is recorded in the table below. If the velocity at $t = 0$ is 6 ft/sec, find the approximate velocity at $t = 18$ using a trapezoidal sum.

Time t , in seconds	0	3	6	9	12	15	18
$a(t)$ (ft/sec ²)	1	4	6	8	7	9	10

Accumulation from a Graph

EX #6: The graph at right shows the rate in gallons per hour, at which storm water is being pumped from an overflowing canal. Find the approximate number of gallons pumped from the canal in the 24-hour period.



Marginal Cost

EX #7: Find the cost to produce 50 widgets if the initial cost is \$1800 and $C'(x) = 155 - 30x + x^2$

Let x be the number of items of a widget produced. Let $C(x)$ be the cost to produce x widgets. $C'(x)$ is called the **marginal cost**, the cost to produce one additional widget if currently x items are produced. That is, $C'(x) \approx C(x+1) - C(x)$



NOTES

8.3

Accumulation Functions in
Applied Contexts Homework

Name _____

Date _____ Period _____

Problems 1 – 5, Describe the meaning of each expression in context to the given scenario.

1. Your new hot tub is being filled with water at a rate of $r(t)$ gallons per minute, where t is the time in minutes. Explain the meaning of the expression.

$$\int_7^8 r(t) dt = 3$$

2. The function $C(t)$ gives the number of spicy chicken sandwiches sold at Fast Chicklet Drive-Thru by time t (in days) for any given month. What does the expression represent?

$$\int_0^7 C'(t) dt$$

3. The number of students exposed to a virus is increasing at a rate of $r(t)$ students per day, where t is the time in days. On day $t = 4$ there were 62 students exposed to the virus. What does the following expression represent?

$$62 + \int_4^{12} r(t) dt$$

4. The download speed of a file from a streaming service is increasing at a rate of $r(t)$ megabytes per second (MBps). Initially the speed is 85 MBps. Explain the meaning of the expression.

$$85 + \int_0^{60} r(t) dt$$

5. The function $b(t)$ gives the temperature, in degrees Fahrenheit, of a birthday cake in time, t in minutes. Describe the meaning of the following.

$$\int_0^{15} b'(t) dt = 118$$

Problems 6 – 7, Given the scenario, find the expression that models each situation.

6. The temperature of your homemade chili is increasing at a rate of $r(t) = 26e^{-0.3t}$ degrees Fahrenheit per minute, where t is the time in minutes. At time $t = 0$, the temperature of the chili is 75°F. What is the temperature at $t = 10$ minutes.

A. $\int_0^{10} r(t) dt$

B. $r(10) - r(0)$

C. $75 + \int_0^{10} r(t) dt$

D. $75 + (r(10))$

7. Randy and Rita Roach live inside the walls of an abandoned house. The population of their roach colony is growing at a rate $r(t)$ roaches per week, where t is time in weeks. At time $t = 3$, the roach colony is 1820 roaches. What is the roach population at $t = 6$?

A. $r'(6) - r'(3)$

B. $\int_3^6 r(t) dt$

C. $1820 + \int_0^6 r(t) dt$

D. $1820 + \int_3^6 r(t) dt$

8. The rate at which customers call to report credit card fraud at a customer service center is modeled by the function $F(t) = 8 + 4 \cos\left(\frac{\pi}{6}t\right)$ for $0 \leq t \leq 60$, where $F(t)$ is measured in calls per minute and t is measured in minutes. To the nearest whole number, how many customers call into the center over the 60-minute period?



9. Transmission fluid is leaking out of a vehicle at a rate of $r(t) = -3(t - 2)$ ounces per hour, where t is the number of hours. How many ounces of fluid leak from the vehicle between $t = 0$ and $t = 3$ hours.



10. Snow is falling steadily at a mountain resort at the rate of $s(t) = \frac{18}{t+3}$ inches per hour; where t is the number of hours since the start of the snowfall. How many total inches of snow fell at the resort between $t = 0$ and $t = 4$ hours?



11. The total revenue from ticket sales for the Junior-Senior Prom as a function of a single ticket's price, x , changes at a rate of $r(x) = 22 - 0.32x$ thousands of dollars per dollar. When $x = 65$, the total expected revenue is 14 thousand dollars. What is the expected revenue if the ticket price is raised to \$75?



13. The Student Council elections were held at school between 7:00 am and 3:00 pm. Students deposited their ballots in a special box located in the media center. The number of entries in the box t hours after 7:00 am is modeled by a differentiable function $E(t)$ for $0 \leq t \leq 8$. The table below shows the number of ballots, at various times t during the day.



t (hours)	0	3	5	6	8
$E(t)$ (ballots)	0	120	382	578	694

- A. At what rate are the ballots being deposited into the box at noon ($t = 5$)? Show your computations that lead to your answer.

- B. Using a trapezoidal sum with four subintervals, approximate the value of the expression below. Then, using correct units in context of the scenario, explain the meaning of the expression.

$$\frac{1}{8} \int_0^8 E(t) dt$$

- C. At 3:00 pm, the student council election committee begins to count the ballots at a rate modeled by the function where $P(t) = \frac{250}{1 + (0.6t - 5)^2}$ ballots per hour for $8 \leq t \leq 12$. According to the model, how many ballots remain to be counted by 6:00 p.m. ($t = 11$)?

Practice Sets

8.3 Accumulation Functions in Context Form C

Name _____

Date _____ Period _____

- The number of households in a local community who have adopted a pet from a shelter is increasing at a rate $r(t)$ households per year, where t is time in years. At $t = 3$ the adoptions were 2846. What does the expression mean?

$$2846 + \int_3^5 r(t) dt$$

Questions 2 – 3: The depth of a chemical is leaking from a storage barrel at a rate of $r(t) = 0.26t + .54$ centimeters per hour, where t is the time in hours. At $t = 0$ the depth of the chemical is at 62 centimeters.

- Write an expression that could be used to find what is the change in the chemical's depth during the 8th hour.

- What is the depth of the chemical in the storage barrel after 12 hours?



8.3 Accumulation Functions in Context Form D

Name _____

Date _____ Period _____

- Jenny has been hiking through Portugal at a rate of $r(t)$ kilometers per hour, where t is time in hours. At $t = 3$ she has hiked 7 km. What does the expression mean?

$$7 + \int_3^6 r(t) dt = 16$$

Questions 2 – 3: Snow is falling on a ski resort at a rate of $r(t) = \frac{12}{t+3}$ inches per hour, where t is the time in hours. At $t = 0$ there are 8 inches of snow.

- Write an expression that could be used to find how much snow fell in the first 3 hours.

- Find the snow if the snow after 6 hours.

8.3 Accumulation Functions in Context Form A

Name _____

Date _____ Period _____

- The population of a beachside resort grows at a rate of $r(t)$ people per year, where t is time in years. At $t = 2$, the resort population is 4823 residents. What does the expression mean?

$$4823 + \int_2^5 r(t) dt = 7635$$

Questions 2 – 3: The temperature of a pot of chicken soup is increasing at a rate of $r(t) = 34e^{-0.8t}$ degrees Celsius per minute, where t is the time in minutes. At time $t = 0$, the soup is 26 degrees Celsius.

- Write an expression that could be used to find how much the temperature increased between $t = 0$ and $t = 10$ minutes.

- What is the temperature of the soup after 5 minutes?



8.3 Accumulation Functions in Context Form B

Name _____

Date _____ Period _____

- The number of residents in a nursing home that have been exposed to the flu is increasing at a rate of $r(t)$ people per day, where t is the time in days. At $t = 5$, there are 18 residents exposed to the flu. What does the expression mean?

$$18 + \int_5^{12} r(t) dt$$

Questions 2 – 3: The popularity of a country music group is increasing at a rate of $r(t) = 1.3e^{0.2t}$ percent of the group's listeners, where t is the time in months.

- Write an expression that could be used to find the percentage of popularity during the sixth month.

- What is the change in the percentage of growth for the group's popularity during the sixth month?



Daily Lesson Quiz