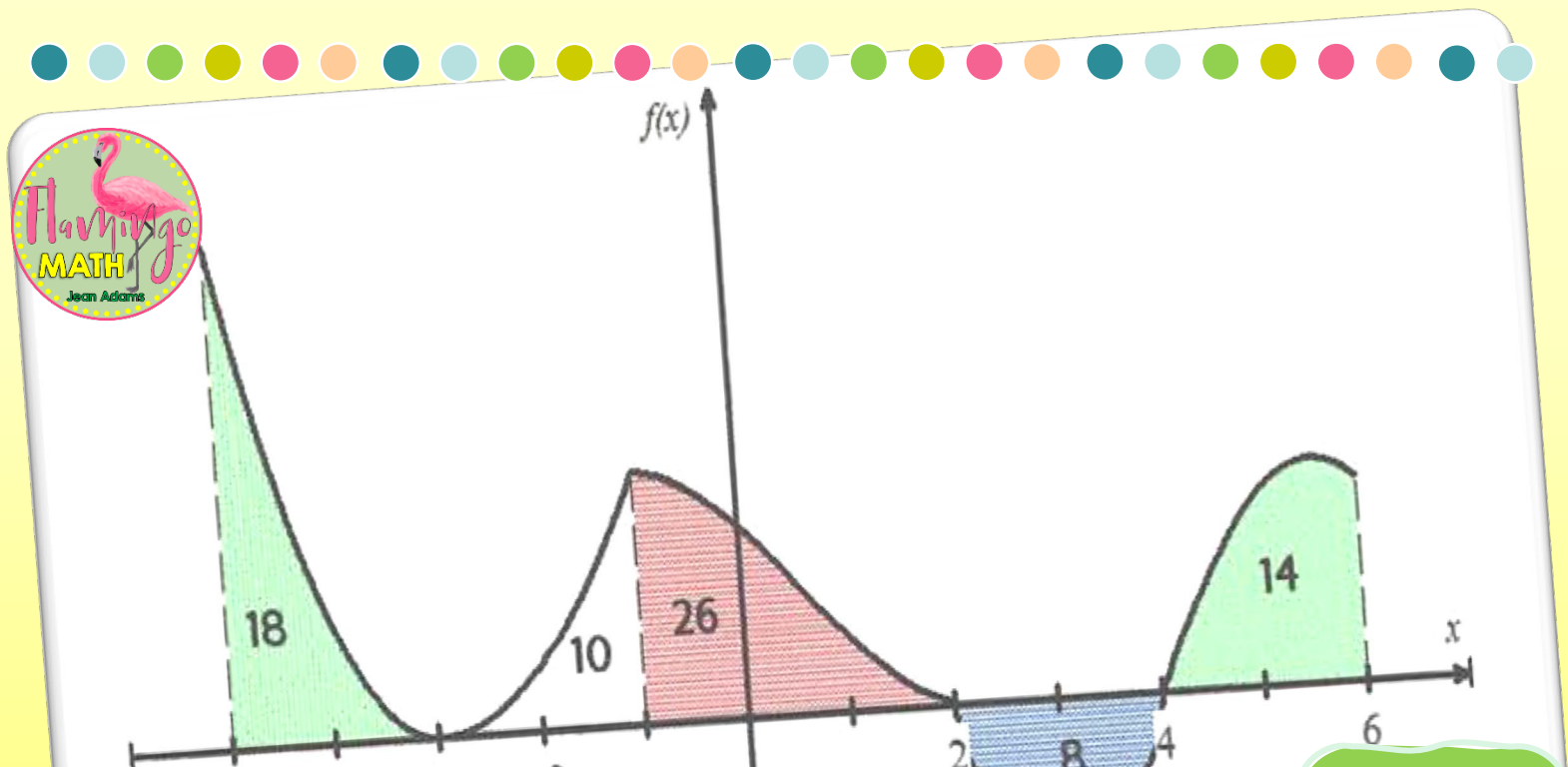


AP[®] Calculus

Accumulation Functions

The Fundamental Theorem of Calculus



Completely Editable

6.3

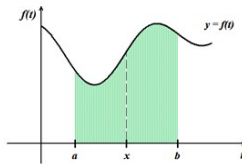
Lesson 6.3: Accumulation Functions

Topic 6.4: The Fundamental Theorem of Calculus and Accumulation Functions Topic 6.5: Interpreting the Behavior of Accumulation Functions Involving Area

Given some function $y = f(x)$ is a continuous function on $[a, b]$, shown below. Suppose we define some function whose area under the curve between point a and some other point, x . We can denote the area under the curve as a definite integral. Let's define this as a new function $F(x)$, where x is in $[a, b]$:

$$F(x) = \int_a^x f(t) dt$$

So, for any value of x between a and b , this new function will give us the area under $f(t)$ between a and x . This is the Fundamental Theorem of Calculus Part I and helps to make the connection between integrals and derivatives. Namely, that every continuous function $f(x)$ has an antiderivative $F(x)$.



Fundamental Theorem of Calculus, Part I

If $f(x)$ is continuous on $[a, b]$ then, $g(x) = \int_a^x f(t) dt$ is continuous on $[a, b]$ and it is differentiable on (a, b) also $g'(x) = f(x)$.

An alternate notation can be seen by taking

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt$$

EX #1: The graph of f is shown below. Let $g(x) = \int_{-6}^x f(t) dt$

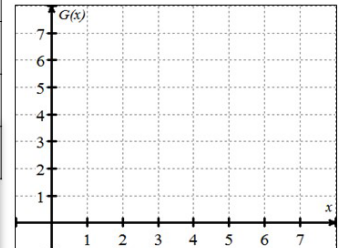
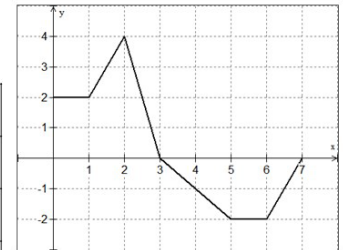
- A. $g(-3)$
- B. $g(0)$
- C. $g(4)$
- D. $g(7)$

EX #2: Let f be the graph of the function shown at right. Where

$$G(x) = \int_0^x f(t) dt$$

A. Complete the table of values for G .

Input	$G(x) = \int_0^x f(t) dt$	Output
0	$G(0) = \int_0^0 f(t) dt$	
1	$G(1) = \int_0^1 f(t) dt$	
2	$G(\quad) = \int_0^{\quad} f(t) dt$	
3	$G(\quad) = \int_0^{\quad} f(t) dt$	
6	$G(\quad) = \int_0^{\quad} f(t) dt$	



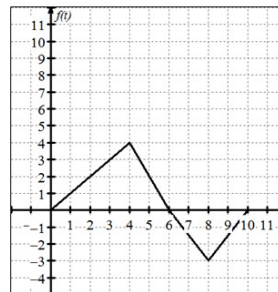
Transforming the Area Accumulation Function by Altering Its Limits of Integration

Let's examine what would happen geometrically to $F(x) = \int_k^x f(t) dt$ if we allow k to take on different values.

EX #3: Given $F(x) = \int_k^x f(t) dt$

A. Complete the table of values for the function F .

x	$F(x)$
0	
1	
2	
3	
4	



- B. What is the domain of F ?
- C. Use the concept of "area accumulation" to complete the table in blue.
- D. Where is the zero of F ?
- E. What is the maximum value of F ?
- F. How does the "total change" in F compare to the total change in f and the x -axis?
- G. What is the range of F ?

EX #4: Now consider the function $G(x) = \int_3^x f(t) dt$

- A. Where is the zero of G ?
- B. What is the domain of G ?
- C. What is the maximum value of G ?
- D. What is the range of G ?

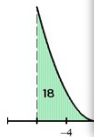
I. Describe the relationship between all $F(x) = \int_k^x f(t) dt$

J. What point must lie on the graph of any given family

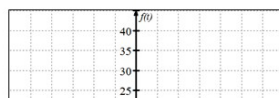
K. As k takes on different values, the functions $F(x) = \int_k^x f(t) dt$ are transformations of one another, each containing the lower bound changed is $F'(x) = f(x)$?

EX #5: Shown below is the graph of the derivative f of F

- A. $F(2) =$
- B. $F(6) =$
- C. $F(-5) =$



- D. Since the area under f is positive on the interval $[-3, -1]$, we know that the graph of F must be _____ on the interval $[-3, -1]$.
- E. Since the area under f accumulates at an increasing rate, we know that the graph of F is _____ on the interval $[-3, -1]$.



Calculus-Based Justification from the graph of $g' = f$

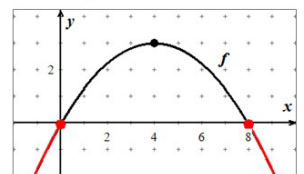
In differential calculus we used the derivative f' to discuss properties of a function f . In integral calculus we need to reverse our thinking. Our new goal is to use derivative functions to talk about antiderivatives. In the example below, we say that g is an **antiderivative of f** . Let's summarize the relationships between the properties of a function and its antiderivative.

EX #6: Given the graph of f where $x = 4$ is the only critical value of f , Let

$$g(x) = \int_0^x f(t) dt$$

A. f is positive on the interval $[0, 8]$, so g is _____ on this interval.

B. f changes sign at _____, so g must have a relative _____ since f goes from _____ to _____.



C. The graph of g is decreasing where f is _____, for values of _____.

D. The graph of g is increasing on _____, so g is _____.

E. The graph of g is concave down where f is _____, for values _____.

F. The graph of g has a maximum at $x =$ _____, so g has an _____.

Reasoning and Justification Summary

When the derivative function, f is ...

The antiderivative

$$g(x) = \int_0^x f(t) dt$$

Decreasing



Concave Down

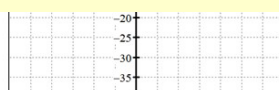


Changes sign
(crosses the x -axis)

Extremum Point

NOTES

x	-5	-3	-1	2	4	6
$F(x)$						



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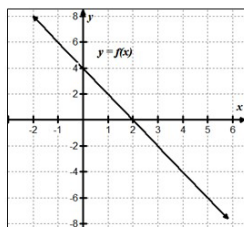
Accumulation Functions
Homework

Name _____

Date _____ Period _____

Show all work.

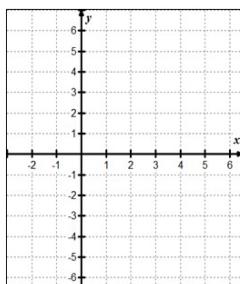
1. Let $F(x) = \int_0^x f(t) dt$, where the graph of f is shown at right. Complete the following chart.



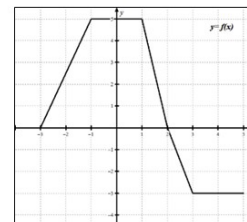
x	-1	0	1	2	3	4	5
$F(x)$							

A. On what interval is f positive? _____B. On what interval is f negative? _____C. On what interval is $F(x)$ increasing? _____D. On what interval is $F(x)$ decreasing? _____

E. Plot the points $(x, F(x))$ on the grid provided. What is the relationship between $f(x)$ and $F(x)$?



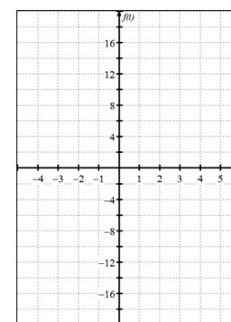
2. Let $F(x)$ be a differentiable function on a closed interval with $F(1) = 13$, whose first derivative $y = f(x)$ is graphed at right. Use the graph of f to complete the table.



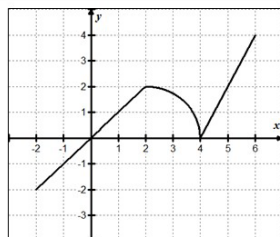
x	-3	0	1	2	3	4	5
$F(x)$			13				

A. On what interval is f positive? _____B. On what interval, is f negative? _____C. On what interval is $F(x)$ increasing? _____D. On what interval, if any, is $F(x)$ decreasing? _____

E. For $-3 \leq x \leq 5$, find the value of x at which $F(x)$ has an absolute minimum. Justify your answer.

F. Graph $F(x)$ on grid at right.

3. Let $F(x) = \int_2^x f(t) dt$. The graph of f on the interval $[-2, 6]$ consists of two line segments and a quarter of a circle, as shown at right.

A. Find $F(0)$ and $F(4)$.

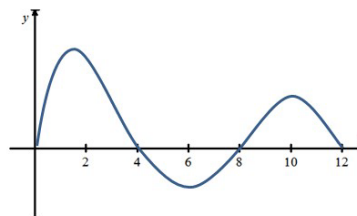
B. Determine the interval where $F(x)$ is increasing. Justify your answer.

C. Find the critical numbers of $F(x)$ and determine if each corresponds to a relative minimum value, a relative maximum value, or neither. Justify your answers.

D. Find the absolute extreme values of $F(x)$ and the x -values at which they occur. Justify your answers.

E. Find the x -coordinates of the inflection points of $F(x)$. Justify your answer.

4. Let $G(x) = \int_0^x f(t) dt$, the graph of f on the interval $[0, 12]$ is shown below.

A. Determine the intervals on which $G(x)$ is increasing. Justify your answer.B. Determine the intervals on which $G(x)$ is decreasing. Justify your answer.C. Find the x -coordinates where $G(x)$ has a local min or max. Justify your answer.D. Find the x -coordinates of the inflection points of $G(x)$. Justify your answer.

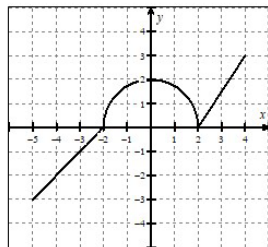
Practice Sets

6.3 Accumulation Functions Form A

Name _____

Date _____ Period _____

Let $F(x) = \int_0^x f(t) dt$. The graph of f on the interval $[-5, 4]$ consists of two line segments and a semi-circle, as shown below.



1. State the critical numbers of $F(x)$

2. Determine the interval(s) where $F(x)$ is increasing. Justify your answer.

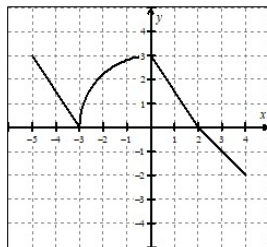
3. Find the absolute extreme values of $F(x)$ and the x -values at which they occur.

6.3 Accumulation Functions Form B

Name _____

Date _____ Period _____

Let $F(x) = \int_0^x f(t) dt$. The graph of f on the interval $[-5, 4]$ consists of two line segments and a semi-circle, as shown below.



1. State the critical numbers of $F(x)$

2. Determine the interval(s) where $F(x)$ is increasing. Justify your answer.

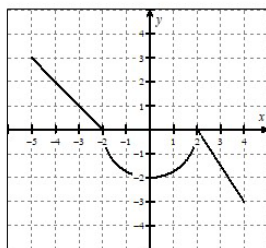
3. Find the absolute extreme values of $F(x)$ and the x -values at which they occur.

6.3 Accumulation Functions Form C

Name _____

Date _____ Period _____

Let $F(x) = \int_0^x f(t) dt$. The graph of f on the interval $[-5, 4]$ consists of two line segments and a semi-circle, as shown below.



1. State the critical numbers of $F(x)$

2. Determine the interval(s) where $F(x)$ is increasing. Justify your answer.

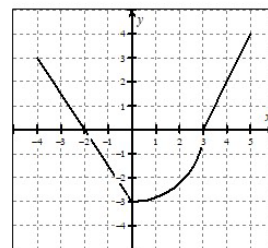
3. Find the absolute extreme values of $F(x)$ and the x -values at which they occur.

6.3 Accumulation Functions Form D

Name _____

Date _____ Period _____

Let $F(x) = \int_0^x f(t) dt$. The graph of f on the interval $[-4, 5]$ consists of two line segments and a semi-circle, as shown below.



1. State the critical numbers of $F(x)$

2. Determine the interval(s) where $F(x)$ is increasing. Justify your answer.

Daily Lesson Quiz