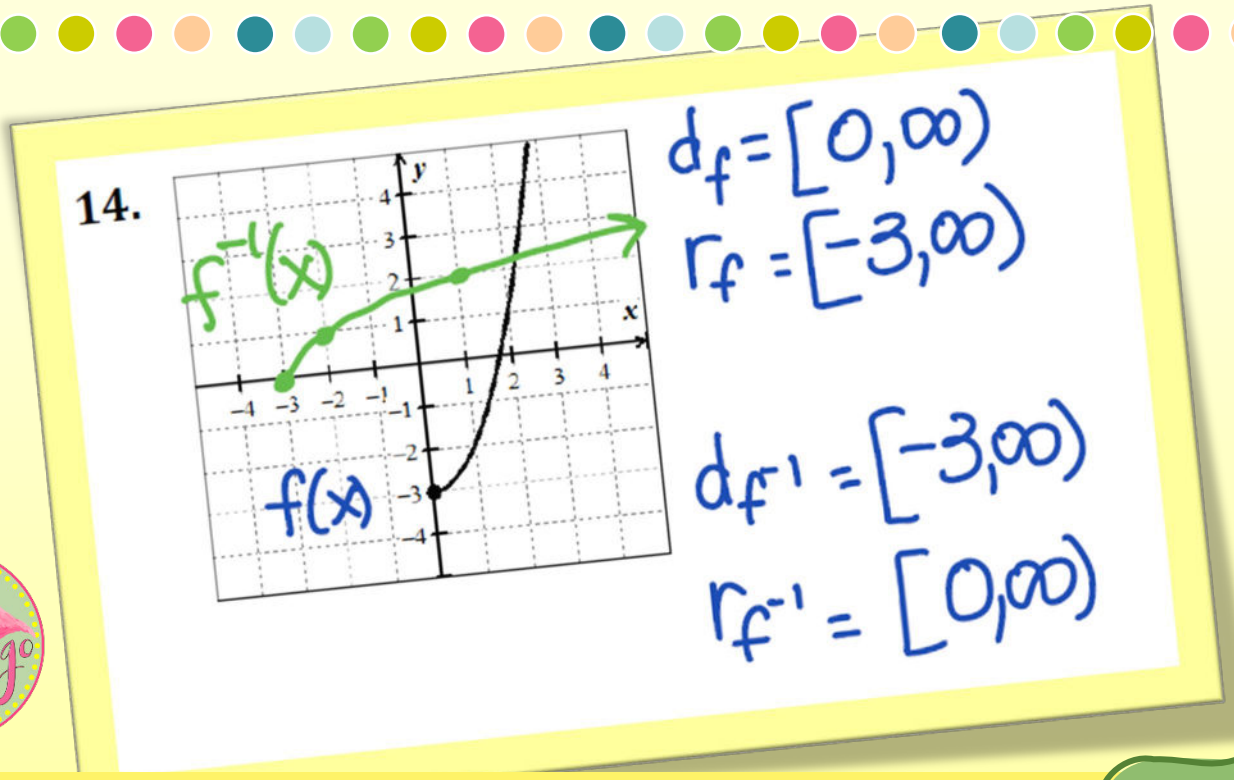
AP® Precalculus

Exponential and Logarithmic Functions



Flamingo MATH
Jean Adams

Homework 2

Completely Editable

Pre-AP PRECALCULUS CURRICULUM

Scope and Sequence

Unit 2: Exponential and Logarithmic Functions

Lesson	Skills & Learning Objectives
[2.1] Change in Arithmetic and Geometric Sequences	▪ Recognize, write and find the *n*th terms of arithmetic sequences ▪ Use arithmetic sequences to model real-world problems ▪ Recognize, write and find the *n*th terms of geometric sequences ▪ Use geometric sequences to model real-world problems
[2.2] Change in Linear and Exponential Functions	▪ Construct functions of the real numbers that are comparable to arithmetic and geometric sequences ▪ Describe similarities and differences between linear and exponential functions ▪ Solve real-world applications using arithmetic and geometric sequences
[2.3] Exponential Functions	▪ Investigate the basic behaviors of exponential functions and their graphs ▪ Analyze exponential graphs for key characteristics such as domain, range, end-behavior, continuity, symmetry, asymptotes, increase and decrease.
[2.4] Exponential Function Manipulation	▪ Use properties of exponents and logarithms to simplify and evaluate expressions ▪ Apply the properties of logarithms to evaluate expressions and graph functions ▪ Understand the characteristics and effects of constants in the exponential model $y = ab^{kx}$

© 2023 Jean Adams, All rights reserved.

Scope and Sequence

Unit 2: Exponential and Logarithmic Functions

Lesson	Skills & Learning Objectives
[2.5] Exponential Function Context and Data Modeling	▪ Construct a model for scenarios that involve proportional output values related to real-world scenarios ▪ Apply exponential models about data sets in contextual scenarios ▪ Construct exponential function models using regression equations and technology
[2.6] Competing Function Model Validation	▪ Find an appropriate regression model for a data set. ▪ Use contextual clues to determine an appropriate model. ▪ Recognize a correct regression model using the graph of the residuals. ▪ Find the residual errors of a model and analyze the findings as underestimating or overestimating the data set in context.
[2.7] Composition of Functions	▪ Perform operations on functions ▪ Find a composition of one function with another function ▪ Rewrite a given function as a composite of two or more functions ▪ Recognize that the composition of functions is not commutative ▪ Recognize the composite function $f(g(x))$ uniquely maps input values of g to output values of f , dependent on the domain restrictions of f and g.
[2.8] Inverse Functions	▪ Verify two functions are inverses algebraically and graphically ▪ Use the Horizontal Line Test to determine if functions are one-to-one ▪ Find inverse functions informally and algebraically ▪ Use function notation to describe real-world scenarios ▪ Analyze and choose the appropriate method to find the inverse of a relation or function

© 2023 Jean Adams, All rights reserved.

Scope and Sequence

Unit 2: Exponential and Logarithmic Functions

Lesson	Skills & Learning Objectives
[2.9] Logarithmic Expressions	▪ Apply the properties of logarithms to evaluate expressions with and without technology ▪ Use properties of exponents and logarithms to simplify and evaluate expressions ▪ Understand each unit of the logarithmic scale is a multiplicative change of the base $10^0, 10^1, 10^2, \ldots$
[2.10] Inverses of Exponential Functions	▪ Use the general form of a logarithmic function to evaluate and graph. ▪ $f(x) = \log_b x$ and $g(x) = b^x$, if $g\big(f(x)\big) = f\big(g(x)\big) = x$ then $f(x)$ and $g(x)$ are inverse functions. ▪ Graphs of inverse functions are reflections across the line $y = x$. ▪ Analyze exponential and logarithmic graphs for domain, range, end-behavior, concavity, asymptotes, increase and decrease. ▪ Determine whether a function behavior is exponential or logarithmic from proportionality of input and output values.
[2.11] Logarithmic Functions	▪ Understand the domain of a logarithmic function in general form is any real number greater than zero, and the range is all real numbers. ▪ Identify key characteristics of logarithmic functions and transformations. ▪ Determine whether logarithmic functions are increasing decreasing, concave up, or concave down. ▪ Logarithmic functions do not have extrema, except on a closed interval, and their graphs do not have inflection points. ▪ Analyze and use limit notation for end behavior.

© 2023 Jean Adams, All rights reserved.

Scope and Sequence

Unit 2: Exponential and Logarithmic Functions

Lesson	Skills & Learning Objectives
[2.12] Logarithmic Function Manipulation	▪ Apply the properties of logarithms to evaluate expressions and graph functions ▪ Use properties of exponents and logarithms to simplify and evaluate expressions ▪ Recognize the graphic translations of logarithms related to the product, power, and change of base property
[2.13] Exponential and Logarithmic Equations and Inequalities	▪ Use properties of exponents and logarithms to solve equations and inequalities ▪ When solving exponential and logarithmic equations analyze for extraneous solutions ▪ Use inverse operations to write exponential and logarithmic equations that reverse the mapping
[2.14] Logarithmic Function Context and Data Modeling	▪ Write a logarithmic function given two ordered pairs or a proportion and a real zero. ▪ Use logarithmic functions to model proportional growth. ▪ Use transformations based on characteristics of a context or data set to model logarithmic behavior. ▪ Use logarithmic regression and technology given a data set. ▪ Use the natural log function to model natural phenomena ▪ Use logarithmic models to predict values for the dependent variable.
[2.15] Semi-log Plots	▪ Use semi-log plots with the y-axis logarithmically scaled that show linear behavior from exponential characteristics. ▪ Apply techniques used to model linear behavior to semi-log graphs. ▪ Construct the linearization of exponential data.

© 2023 Jean Adams, All rights reserved.

2.10

Inverses of Exponential Functions Practice Set 1

Name _____

Date _____ Period _____

Problems 1 – 2, In the tables shown below, the value for b is the same positive constant.
Table I contains outputs of the function $f(x) = b^x$ for some x values.
Table II contains outputs of the function $g(x) = \log_b x$ for some x values.
Find the values for a, b, c, and d without using a calculator.

1.

Table I				
$\boldsymbol{x}$	0.631	1.262	1.465	1.631
$\boldsymbol{b^x}$	2	4	5	6

Table II					
$\boldsymbol{y}$	a	3	4	$2c$	$4d$
$\log_{\boldsymbol{b}} \boldsymbol{y}$	0	1	1.262	1.465	1.631

2.

Table I				
$\boldsymbol{x}$	0.792	1.161	1.292	1.5
$\boldsymbol{b^x}$	3	5	6	8

Table II					
$\boldsymbol{y}$	a	4	$4c$	7	$12d$
$\log_{\boldsymbol{b}} \boldsymbol{y}$	0	1	1.292	1.404	1.5

Problems 3 – 6, Find the inverse of each function.

3. $y = log_3 x$

4. $y = log(x + 2)$

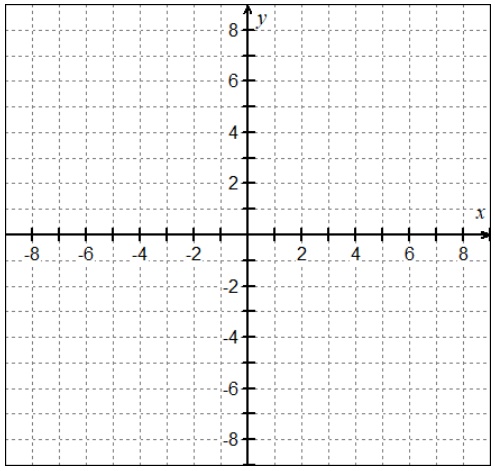
5. $y = log_2(2x)$

6. $y = log_{\frac{1}{2}} x$

Problems 7 – 8, Use the given function $y = f(x)$ to:

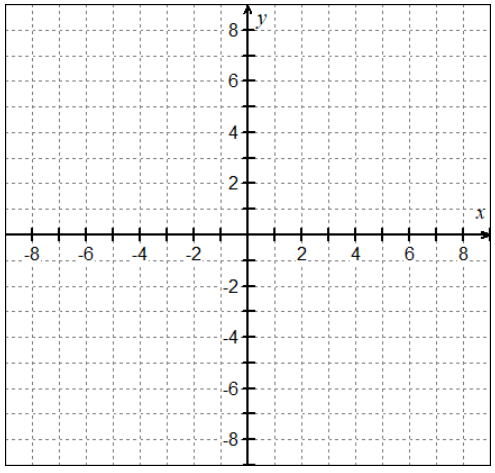
A. Find the domain of f **B.** Graph f.
C. From the graph, determine the range and any asymptotes of f.
D. Find f^{-1} , the inverse of f.
E. Graph f^{-1}.
F. Find the domain and range and any asymptotes of f^{-1}.

7. $f(x) = 2^{x-3}$



Domain	Range	Asymptotes

8. $f(x) = \log_2(2x) + 1$



Domain	Range	Asymptotes

Free Response Question

14. The table below is a representation of values for the function $f(x) = e^{x-3} + 2$.

x	-6	-1	0	1	4	7
$f(x)$	2.0001	2.018	2.049	2.135	4.718	56.598

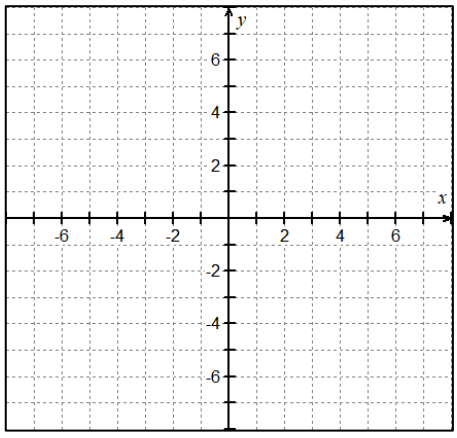
A. What is the equation of the horizontal asymptote for $f(x)$. Justify your answer.

B. Does the graph of $f(x)$ lie above or below the horizontal asymptote. Explain your reasoning.

C. Let $g(x)$ represent the inverse of $f(x)$. Find the equation of the inverse function $f^{-1}(x) = g(x)$. Show your work to support your answer.

D. What is the vertical asymptote of $y = g(x)$? Justify your answer.

E. Sketch both functions on the grid provided.
State the domain and range of both $f(x)$ and $g(x)$.



2.8

Inverse Functions
Practice Set 1

Name _____

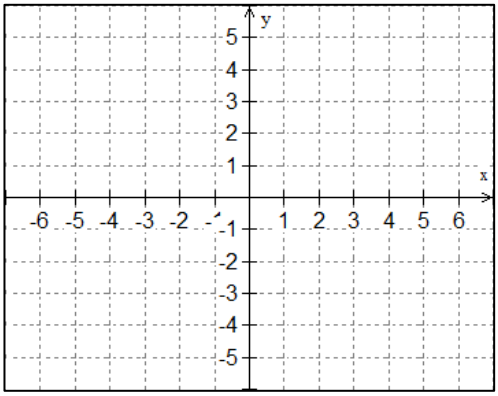
Date _____ Period _____

Problems 1 – 2, find the inverse of each relation. Graph the given relation and its inverse.

1.

x	y
0	-3
1	-1
2	1
3	3

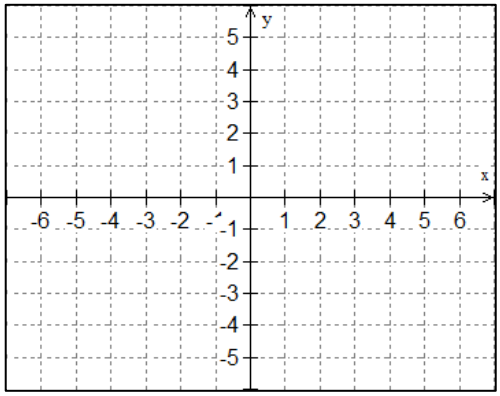
x	y



2.

x	y
-6	3
-1	2
2	1
3	0

x	y



Problems 3 – 6, find the inverse of each function, if it exists.

3. $y = 2x - 5$

4. $y = 3 - 2x^2$

5. $y = \sqrt{x - 2}$

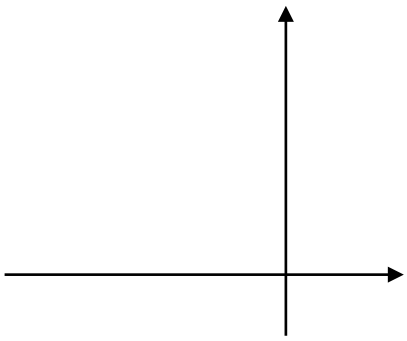
6. $y = (x - 1)^2$

20. $h(x) = f(x) + 2$

$\boldsymbol{x}$	-3	-2	-1	0	1
$\boldsymbol{f(x)}$					
$\boldsymbol{h(x)}$	66	18	6	3	2.25

Problems 21 – 22, graph the function, label intercept(s), asymptotes, and analyze completely.

21. $f(x) = 4e^x$



Domain: Range:

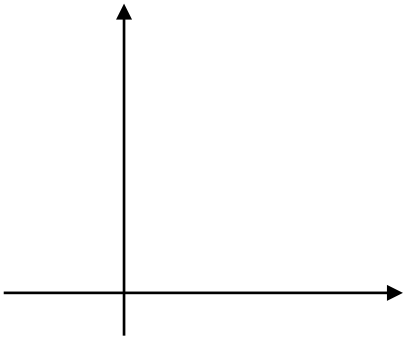
Continuity: Increase / Decrease

Extrema: Asymptotes:

End-behavior:

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$

22. $g(x) = 3\left(\frac{1}{2}\right)^x$



Domain: Range:

Continuity: Increase / Decrease

Extrema: Asymptotes:

End-behavior:

$\lim_{x\to-\infty} g(x) =$ $\lim_{x\to\infty} g(x) =$

Problems 23 – 24, use the table and given information to answer the questions.

23. An exponential function of the form $F(x) = a \cdot b^x + c$ is shown in the table below.

$\boldsymbol{x}$	-8	-6	-3	-1	1	3	6
$\boldsymbol{F(x)}$	4.9997	4.9973	4.9259	4.3333	-1	-49	-1453

A. Describe the end behavior of the function using limit notation.

B. What is the equation of the horizontal asymptote?

C. What conclusion can be made about the value of a? Why?

D. What is the value of c ? Give a reason.

24. An exponential function of the form $G(x) = a \cdot b^{-x} + c$ is shown in the table below.

$\boldsymbol{x}$	-8	-6	-3	-1	1	3	8
$\boldsymbol{G(x)}$	487	92	17	4	3.25	2.477	2.003

A. Describe the end behavior of the function using limit notation.

B. What is the equation of the horizontal asymptote?

C. What conclusion can be made about the value of a? Why?

D. What is the value of c ? Give a reason.