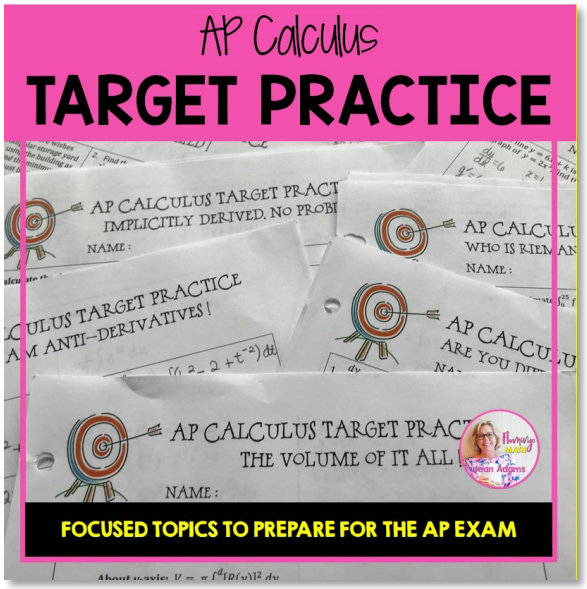


AP CALCULUS AB-BC TARGET PRACTICE

This resource is set of 22 targeted topics for focused preparation on the AP Exam.

As we approach the AP Calculus AB-BC Exam, many students need a topical approach to the types of questions they will be asked to complete on the AP Calculus AB-BC Exam. In this resource, you will find the following 22 concepts for students to "aim for success."

- ❑ Know Your Limits
- ❑ This Seems So Derivative To Me
- ❑ How Extrema Can You Get?
- ❑ Extrema Applied
- ❑ Implicitly Derived, No Problem!
- ❑ Connecting F, F', and F'' Together
- ❑ Who Is Riemann, What Does He Sum?
- ❑ I Am Anti-Derivatives!
- ❑ Are You Differential About Equations?
- ❑ The Volume Of It All!
- ❑ How Much Did You Accumulate?
- ❑ Did You Say Volume with a Shape?
- ❑ A Particle is Moving . . . ?
- ❑ I'm Growing Exponentially!
- ❑ L'Hospital's Rule
- ❑ Sigma Notation and Riemann Sums
- ❑ Convergence Tests (BC Topic)
- ❑ Advanced Integration (BC Topic)
- ❑ Arc Length and Improper Integrals (BC Topic)
- ❑ Parametric and Vector-Valued Functions (BC Topic)
- ❑ Maclaurin and Taylor Polynomials (BC Topic)
- ❑ Power Series and Intervals of Convergence (BC Topic)

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AP CALCULUS TARGET PRACTICE

KNOW YOUR LIMITS!

NAME:

1. $\lim_{x \to 2} \frac{x^3 + 3x^2 - 4x - 12}{x - 2}$	2. $\lim_{x \to 9} \frac{9 - x}{\sqrt{x} - 3}$
3. $\lim_{x \to 0} \frac{4x^2}{1 - \cos 2x}$	4. $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$
5. $\lim_{\theta \to 0} \frac{\tan\theta \cos\theta}{\theta}$	6. $\lim_{\theta \to -\infty} \frac{\sin\theta}{\theta}$
7. $\lim_{x \to \infty}$	
9. $\lim_{t \to 0}$	
11. $\lim_{x \to 5^-}$	
13. $\lim_{h \to 0}$	

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AP CALCULUS TARGET PRACTICE

THIS SEEMS SO DERIVATIVE TO ME!

NAME:

1. $y = 6x^{\frac{2}{3}} - 4x^{\frac{1}{2}} - 3$	2. $f(x) = \sqrt{x} + \frac{2}{x}$
3. $h(x) = \frac{3 - x}{2x + 1}$	4. $f(x) = \ln\frac{e^x}{e^x}$
5. $y = \frac{4x^6 - 1}{x^2}$	6. $y = \frac{x}{4x^4}$
$f(\theta) = 5\theta \sec(5\theta)$	
9. $\frac{d}{dx}[\ln(x^2 +$	
$v(t) = \sqrt[4]{2t^3 +$	
13. $y = 5e^{\cos x}$	

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AP CALCULUS TARGET PRACTICE

EXTREMA APPLIED!

NAME:

1. The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining sides of the yard.	2. Find the length and width of a rectangle that has an area of 200 square feet and whose perimeter is a minimum.
3. **(Calculator.)** An open box is constructed from cardboard by cutting out squares of equal size from the corners and then folding up the sides. If the cardboard is 5 inches by 10 inches, determine the volume of the largest box which can be constructed. Round your answer to three decimal places.	4. Lionel is 5 feet tall and is walking away from a light hung 20 feet from the ground, at a rate of 6 ft/sec. How fast is his shadow lengthening?

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AP CALCULUS TARGET PRACTICE

HOW EXTREMA CAN YOU GET?

NAME:

Find the critical numbers of the following:

1. $f(x) = x^3 - 2$	2. $f(x) = \cos\theta + \sin\theta$, 2

Find the relative maximum and minimum values on the given interval.

3. $f(x) = 3x$ [illegible]	4. $f(x) = x^3 - 27x$; $[-3, 4]$

4. Find the x-coordinate of the point c, that is guaranteed by the Mean Value Theorem on the interval $[1, 5]$ for $f(x) = 4 + \sqrt{x - 1}$

5. If $f(x) = x^3 - x^2 - 2x$, on $[-1, 2]$ determine whether Rolle's Theorem can be applied. If it can, find all values of c that satisfy the conclusion of the theorem.

Find the intervals for which the functions are increasing or decreasing. Justify your answers.

6. $f(x) = \frac{x^2}{x^2 - 9}$	7. $f(x) = \frac{1}{3}x^3 - 5x$

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PREVIEW

5. At a cement plant, sand is stored in a container called a hopper. The shape of the container is an inverted cone. The height is 18 feet and the radius is 12 feet. If the hopper is full and sand is emptied at the rate of 4 ft^3/min, how fast is the level of sand dropping when the sand level is 6 feet high? $V = \frac{1}{3}\pi r^2 h$

6. Air is pumped into a spherical balloon at the rate of 100 cm^3/sec. How fast is the diameter increasing when the radius is 5 cm? $V = \frac{4}{3}\pi r^3$

7. A t
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AP CALCULUS TARGET PRACTICE
IMPLICITLY DERIVED, NO PROBLEM!

NAME:

Calculate the derivative with respect to x.

1. $3y^3 + x^2 = 10$

2. $x^4 + y^4 = 2$

3. $y^4 - 2y = 4x^3 + x$

4. $\frac{y}{x} = \frac{x}{}$

5. $\sqrt{y^2} =$

7. $e^{xy} = \sin(y$

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9. $\ln x + \ln y = x - y$

10. $\ln(x^2 + y^2) = x + 6$

Find an e... of the ta... line at ...e given po... .

11. $x^2 - y^4 = 1$ at $(-1, 1)$

12. x $x^2y^2 = 5$ at $(2,1)$

13. $+ \sin$ $= xy^2 + 1$ at $(1,0)$

14. e $^{-y} = \frac{x^2}{y}$ at $4)$

15. $\sqrt{x} + \sqrt{y} - 1 = y$ at $),4)$

16. $x^2 + y^2 = 25$ at $(-3,4)$

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AP C...CULU... TARGET PRACTICE
CONN...CTING F... AND F' TOG...THER!

NAME:

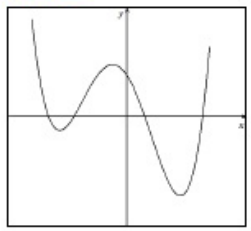
1. If the line $= 6x + k$ i... ngent to the graph of y $2x^3$, find t... value(s) of k.

2. If the posit... function of a p... icle $x(t) = \frac{1}{3}t^3 - ...^2 + 4$, find th... veloci... and position of ... particle w... its acceleration is 0.

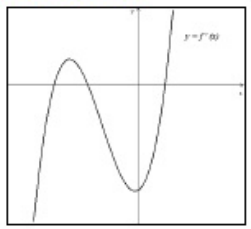
3. Find the coor... es of each point on the graph ... $x^4 - 3x + 10 = 0$ at which ... ngent line is vertical. Write an equation of each vertical tangent line.

4. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is $(t) = t^3 - 3t^2 + 12t + 4$ is

5. This is the graph of $f(x)$. Sketch the derivative.



6. This is the graph of the derivative. Sketch the graph of $y = f(x)$.



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AP CALCULUS TARGET PRACTICE

WHO IS RIEMANN, WHAT DOES HE SUM ?

NAME :

1. Use a **right Riemann sum** to estimate $\int_0^{25} f(x)dx$, with five rectangles.

x	0	5	10	15	20	25
$f(x)$	98	80	68	56	51	43

2. Use a **left Riemann sum** to estimate $\int_0^{25} f(x)dx$, with five rectangles.

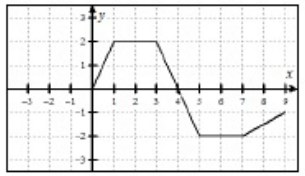
x	0	4	12	16	18	25

3. Use

4. The
mid

2014 F

5. The graph of f is shown at right.
$F(x) = \int_0^x f(t)\,dt$, fill in the values for $F(x)$ in the table.



x	0	1	2	3	4	5	6			9
$F(x)$										

6. Suppose the velocity of a mov ject is given by $v(t) = t^2$ 0, wh $v(t)$ is in mi per hour and t is in hours n point sums and three rectan of eq width to find approximati e de ite integral $\int_1^4 v(t)\,dt$

7. Given $f(x) = 2x$ x
width. Find the a
definite integral. C

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I AM ANTI-DERIVAT !

NAME :

1. $\int \frac{2x^2 - x + 3}{\sqrt{x}}\,dx$

2. $\int \left(t - \frac{1}{t}\right)^2 dt$

3. $\int \frac{x^3}{x+} \, dx \; ; x \neq -2$

4. $\int \frac{(\sqrt{x}+2)}{x^3} \, dx$

5. $\int \frac{1}{\sin^2} \, dx$

6. $\int (2 \quad 2\tan^2\theta)\,d\theta$

7. $\int x^2\sqrt{2} \quad +5 \, dx$

8. $\int \frac{\sin 2}{\sqrt{1-\cos}} \, d\theta$

d the definite integral each fun on.

9. $\int \frac{x-3}{\sqrt{x}} \, dx$

10. $\int |2x$

11. $\int \frac{x}{\sqrt{+9}} \, dx$

12. $\int_0^{\pi/2} 3\sin\left(\frac{\theta}{2}\right) d\theta$

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Fi the indef te integral of transcendent functions.

13. $\int e^{-4x}\,dx$

14. $\int x^2 e^{3x^3}\,dx$

15. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

16. $\int \frac{x-2}{x^2-4x+5} \, dx$

17. $\int \quad)dx$

18. $\int \frac{2^x}{2^x+1}\,dx$

19. $\int 3^{\cos x}\sin x\,dx$

20. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

21. $\int \frac{4x}{x^2-9}\,dx$

22. $\int \frac{x^2+7}{x-2}\,dx$

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AP CALCULUS TARGET PRACTICE

ARE YOU DIFFERENTIAL ABOUT EQUATIONS ?

NAME :

1. $\frac{dy}{dx} = \frac{x}{y}$	2. $\frac{dy}{dx} = \frac{x^2 - 3}{2y^2}$
3. $(3 + x)\frac{dy}{dx} = 5y$	4. $(1 + 9x^2)\frac{dy}{dx} = 3$
5.	
7.	
9.	

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11. $\frac{dx}{dt} = \frac{e^t}{x}$; $x(0) = -1$	12. $\sqrt{y}\frac{dy}{dx} = \sqrt{x}$; $y(1) = 4$
13. Yiluvia is a radioactive substance that has a half-life of 162 years. How long will it take 74 gram sample to degrade to 22 grams? Round your answer to the three decimal places.	14. The number of bacteria in a culture is increasing according to the law of exponential growth. There are 150 bacteria in the culture after two hours and 380 bacteria after four hours. Find the initial population. Round your answer to the ne[illegible]whole number.
15. [illegible] (1, 9) lies on [illegible] graph of an equation $y = f$ where $x \geq 0$ a[illegible] when $x = 0$.	16. The number of E. co[illegible]cteria grows
17. CALCULATO[illegible] culture is grow[illegible] of time. At $t =$ [illegible] is 5,200. Find t[illegible]	

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AP CALCULUS TARGET PRACTICE

THE VOLUME OF IT AL[illegible]

NAME :

Volume By Disk Method: $V = \pi[$(radi[illegible]$)^2 \cdot$ (thickness)

About *x*-axis: $V = \pi \int_a^b [R(x)]^2 dx$

About *y*-axis: $V = \pi \int_c^d [R(y)]^2 dy$

Volume B[illegible]ner Method: $V = \pi[$(outer radius$)^2 -$ (inner radius$)^2] \cdot$ (thick[illegible]ess)

About *x*-axis: $V = \pi \int_a^b ([R(x)]^2 - [r($[illegible]$)]^2)\, dx$

About *y*-axis: $V = \pi \int_c^d ([R(y)]^2 - [r($[illegible]$)]^2)\, dy$

Sketch each region *R* bounded by the graphs of the equations, and find the volume of the solid generated if *R* is revolved about the indicated axis.

1. $y = \sqrt{x}$; $x = 4$; $y = 0$; *x*-axis

2. $y = 2 - x^2$; $y = 1$; $x = 0$; *y*-axis

3. $y = \sqrt{x + 2}$; $y = x$; $x = 0$; *x*-axis

4. $y = x^3$; $y = [illegible]$; $x = 2$; *x*-axis

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5. $y = x^2 + 1$; $y = [illegible] + 3$; *x*-axis

6. $y^2 = x$; $2y = [illegible]$; *y*-axis



7. $y = x^2$; $y = 0$, $[illegible]$; about the [illegible] $x = 2$

8. $y = 3 - x^2$; $y = 2$; about the line $y = 2$



9. $x = \sqrt{1 + y}$; $y = 2$; $x = 0$; *y*-axis

10. $y = \frac{1}{x}$; $x = 1$; $x = 3$; $y = 0$; *x*-axis



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AP Calculus Target Practice

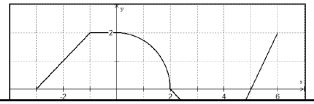
How Much Did You Accumulate?

Name:

$$F(x) = \int_0^x f(t)\,dt$$

You are finding the accumulated area under the curve f starting at zero (or some other value) and going to the value of x. The variable t is called a dummy variable. *It is important that you understand this is a function of x.*

These types of problems are usually associated with particle motion. They go hand-in-hand with finding absolute max/min problems. When f represents velocity, the accumulation function will represent the displacement (or distance) of the particle from $t = 0$ to $t = x$



Use the graph for $y = f(x)$, shown at right, to answer questions 1 – 12. The graph of f consists of l[illegible]

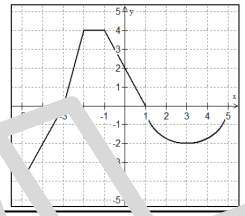
1. $\int_0^2 f(x)\,dx$
3. $\int_2^5 f(x)\,dx$
5. $\int_2^6 f(x)\,dx$
7. $\int_{-2}^2 f(x)\,dx$
9. $\int_4^{-1} [illegible]\,dx$
11. $\int_0^0 |f(x)|$

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If $\int_{-4}^{-1} f(x)\,dx = -7$, $\int_{-1}^{6} f(x)\,dx = 3$ and $\int_4^6 f(x)dx = 6$, find the following:

13. $\int_6^{-4} f(x)dx$ **14.** $\int_{-1}^{4} f(x)\,dx$ **15.** $\int_{-1}^{6} (-4f(x) + 5)\,dx$

16. A graph of f is made up of lines and a semi-circle as shown at right. Let $f(x) = \int_{-1}^{x} f(t)\,dt$.



A.) Find $F(5)$

B.) Find $F(-5)$

$(x) = \int_0^x f(t)$

hown at right, repr

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x	
$F(x)$	
$F'(x)$	

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AP Calculus Target Practice

Did You Say Volume with a Shape?

Name:

1. The base of a solid is the region bounded by the graphs of $x = y^2$ and $x = 4$. Each cross sect[illegible] perpendicular to the x-axis is a square[illegible] the volume of the solid.

2. [illegible]se of a solid [illegible]e region boun[illegible]d by the gr[illegible] $f x = y^2$ and [illegible]= 4. Each cro[illegible] section pe[illegible]n[illegible]ular to the x-[illegible]s is a triangl[illegible]f altitude 2. F[illegible]d t[illegible]volume of the [illegible]lid.

3. Th[illegible]base of a solid is the re[illegible] bounded by the gra[illegible]s of $x = y^2$ and $x = 4$. Ea[illegible]cross section perp[illegible]dicular to the x-axis is a se[illegible]circle. Find the v[illegible]me of the solid.

4. The b[illegible]e of a [illegible] the r[illegible]ion bounded by th graphs [illegible] x^2 and $x = $[illegible] Each cross section perpendicular to the x-axis [illegible]n equilateral triangle. Find the volume of [illegible] solid.

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5. The base of a [illegible]olid is the region bounded by [illegible]raphs of $x =$ [illegible] and $x = 4$. Each cross sectio[illegible] [illegible]rpendicular [illegible] the x-axis is a rectangle of h[illegible]ght 2. Find t[illegible] volume of the solid.

6. Find[illegible]e v[illegible]me of the solid w[illegible] base is bou[illegible]ed by the [illegible]rcl[illegible]$^2 + y^2 = 9$. Each [illegible]ss section perpen[illegible] to the x-axis is an i[illegible]sosceles right triangle, [illegible]he hypotenuse on the base.

7. [illegible]base of a solid is the region bounded by the graphs of $y = \sqrt{x}$ and $y = 4$. Each cross section perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base. Find the volume of the solid.

8. The base of a solid is the region bounded by the graphs of $y = \sqrt[3]{x}$ and $y = 2$. Each cross section perpendicular to the y-axis is a square. Find the volume of the solid.

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PREVIEW

AP Calculus Target Practice

A Particle is Moving . . . ?

Name:

Position (Distance), Velocity, Acceleration

Position Function: $s(t)$ or $x(t)$ motion along the *x*-axis or $y(t)$ motion along the *y*-axis.

Average Velocity: $m_{sec} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$

Instantaneous Velocity: $v(t) = s'(t) = \frac{ds}{dt}$

- Motion to the right or up is considered positive → , then $v(t) > 0$ and distance *s* is increasing
- Motion to the left or down is considered negative ← , then $v(t) < 0$ and distance *s* is decreasing.

Acceleration: $a(t) = v'(t) = \frac{dv}{dt}$ or $a(t) = s''(t) = \frac{d^2s}{dt^2}$

- If $a(t)$ and $v(t)$ have same signs, then speed of *P* is increasing or speeding up
- If $a(t)$ and $v(t)$ have opposite signs, then speed of *P* is decreasing or slowing down.
- If *s* is a continuous function of *t*, then *P* reverses direction whenever *v* is zero and *a* is different from zero. NOTE: zero velocity does not necessarily imply a reversal in direction.

Instantaneous speed: $|v(t)|$

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AP Calculus Target Practice

I'm Growing Exponentially!

Name:

KNOW THIS BACKWARDS AND FORWARDS

"The rate of change of some quantity *y* is directly proportion
can be translated in three ways:

1.) $\frac{dy}{dt} = ky$ 2.) $y = Ce^{kt}$ 3.) $y = y_0e^{kt}$

k is sometimes called th th or decay co ant.

The process to follow . solving such pr ms is a ays the sa :

$\frac{dy}{dt} = ky$

$\frac{dy}{y} = k\,dt$ 1. Separate th bles.

$\int \frac{dy}{y} = \int k\,a$ Integrate b h sides.

$\ln|y| = kt + C$ 3. erform the egration. lace +C on e ri side.

$= e^{kt+C}$ 4. ite this exp entially.

$y = e^{} + C$. D the absol value s e raised t any po r >0.

$y = e^{k} \cdot e^{C}$ Spl he right sic two factors.

$= Ce^{k}$ e^C is mply a constant.

$= y_0$ When $= 0$, $y = C$. We call constant y_0

1. The mount of bacteria in a cultur t any time i reases at a rate proportion the amount bacteria pres t. There are 500 bacteria er the first day d 800 bacteria after the thi ay. Approxi tely how many cteria were present in ally? *Round your swer to the rest whole num*

2. The population a country i rowing at a rate proportiona population. If the growth rate per year is 2.3% of t current po lation, how long will it take for the population to double?

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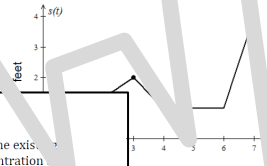
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4. A particle moves along a straight line is given by the position function $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(A) Find all *t* for which the velocity is increasing.
(B) Find where the speed of the particle is increasing.
(C) Find the speed when $t = 1.5$
(D) Find the total distance traveled between $t = 0$ and $t = 4$.

5. A particle moves in a straight line with velocity $v(t) = 4t^2$. How far does the particle ove betwee times $t = 1$ and $t = 2$?

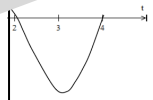
6. T aph of the position fu tion of a moving particle is shown right.



(A) What is the pa position at $t =$

3. he concentration of a me in injec d into the bl dstream drops at a rate propor al to the exis concentration. If the fact of proportio ity is 20% p hour,in how many hours will th oncentration -tenth the initial co ntration?

4. he change of rate of w es N in a po lation is dire y proportio to 650 . hen $= 0$, the populatio s 320, and w en $t = 2$, the pulation has ed to 438. F the wolf pulation after 5 years.



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5. Radiu 226 has a half-life of 1612 years. How long, to the nearest year, will it take 64 grams to decay t ams?

6. Suppose $P(t)$ denotes the size of an animal population at time *t* and its growth is described by the differential equation $\frac{dP}{dt} = 0.002P(1000 - P)$. When is the population growing the fastest?

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PREVIEW

AP Calculus Target Practice
L'Hospital's Rule

Name:

L'Hospital's Rule states that if $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ then $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$

NOTE: Repeat as needed until you can assign a definite value to the expression such as, any real number, or that limit does not exist because there is unbounded behavior toward $\pm\infty$.

Find each limit or state the reason why the limit does not exist.

1. $\lim_{x\to -2}\frac{x+2}{\sqrt{x^2+5}-3}$

2. $\lim_{x\to 3}\frac{x^3-2x^2-3x}{x^3+4x^2-17x-12}$

3. $\lim_{x\to 1}\frac{\ln x^2 - 3x + 3}{e^x - e}$

4. $\lim_{x\to 0}\frac{x^2\sin x - 5\cos x + 5}{x^2}$

5. $\lim_{x\to 2}\frac{4-x^2}{\sin \pi x}$

6. $\lim_{x\to 0}\frac{e^x - x - 1}{\cos x - 1}$

7. $\lim_{x\to\infty}\frac{7x^2+4x-5}{3-4\ldots}$

8. $\lim_{x\to\infty}\frac{2x^2-4}{e^{bx}}$

9. $\lim_{x\to\infty}\frac{5^x}{\ldots 3}$

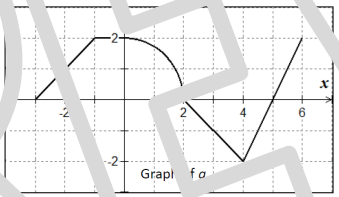
10. $\lim_{t\to -2}\frac{\int_2^t x^3\,dx}{t+3t}$

Questions 11 – 12

A continuous function g is defined on the closed interval $-3 \le x \le$. Let f be the function given by

$(x) = \int_0^x \ldots (t)\,dt$

Also, initial $f(0)$ 2.



Graph of g

Use the graph to evaluate each limit.

11. $\lim_{x\to -1}\frac{f(x)}{x^2-1}$

12. $\lim \frac{2(\ldots)}{\sqrt{x}\ \ \sqrt{5}}$

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AP Calculus Target Practice
Sigma Notation and Riemann Sums

Name:

Calculating Area with a Riemann Sum
NOTE: i or k, either way, this is just a variable to indicate the individual rectangles

$\sum_{k=1}^{n}[f(x_k)\cdot\Delta x_k]$ *or* $\sum_{i=1}^{n}[f(x_i)\cdot\Delta x_i]$

Calculate the ***width*** of one ...ngle

$\Delta x = \frac{b-a}{n}$

Identify t... ***...ries*** of the interval

$[a, \ldots] = [a, a \ldots]$

Write the sum o... te...ns, beginni... at $k = 1$, all the way to $k =$

$\sum_{i=1}^{n}[\Delta x \cdot f(\ldots + \ldots \Delta x)]$

Now, add ...em together ... find the a... between ...he function ...nd the x-ax...

$\lim_{n\to\infty}\sum_{k=\ldots}^{n}[\Delta x \cdot f(a + \ldots k\Delta x)]$

E...ress the a... under the ...aph as a limit using summ...ion notation. Do ...t evaluate.

1. $\int_{\ldots}^{\pi/2} \cos x \ldots$

$\int_1^{\ldots} 3\sqrt{x}\,dx$

3. $\int_2^{\ldots} \ldots(x)\,dx$

4. $\int_{\ldots}^{3} \ldots$

Let n be a ...sitive integ... write each ...it as an integral.

5. $\lim_{n\to\infty}\frac{2}{n}\left[\frac{1}{\ldots(2/n)} + \frac{1}{1+(4/n)} + \cdots + \frac{1}{1+\ldots}\right]$

6. $\lim_{n\to\infty}\frac{\pi}{n}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\ldots}{n}\right) + \cdots + \sin\left(\frac{n\pi}{n}\right)\right]$

Write each area represented by the limit as a definite integral. Evaluate each integral.

7. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{n}k\right)\cdot\cos\left(\frac{\pi}{n}k\right)\cdot\left(\frac{\pi}{n}\right)$

8. $\lim_{n\to\infty}\sum_{k=1}^{n}\left[2+\frac{3}{n}k\right]^2\left(\frac{3}{n}\right)$

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PREVIEW



AP Calculus BC Target Practice

Convergence Tests

Name:

Summary of Tests for Sequences and Series

Test	Series	Converges	Diverges	Comments
Nth Term	$\sum_{n=0}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	NOT for convergence
Geometric Series (GST)	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert > 1$	
p-Series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	
Alternating Series Test (AST)	$\sum_{n=1}^{\infty} (-1)^n a_n$ $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	$0 < a_{n+1} \leq a_n$ $\lim_{n\to\infty} a_n = 0$		
Integral Test f **is positive, continuous, and decreasing**	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	$\int_1^{\infty} f(x)\,dx$ converges, then series converges	$\int_1^{\infty} f(x)\,dx$ diverges, then series diverges	
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$ or $= \infty$	
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \lvert a_n\rvert > 1$ or $= \infty$	
Direct Comparison Test (DCT)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} a_n$ Diverges	
Limit Comparison Test (LCT)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Do the following diverge, converge, or can't be determined? If they converge, find the sum.

1. $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+3}\right)$
2. $\sum_{n=1}^{\infty} \frac{5}{n}$
3. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{5^n}$
4. $\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$

When using the Ordinary Comparison Test (OCT), we usually use either a series for which we determine whether the series converges or diverge.

5. $\sum \frac{2^n}{1+3^n}$
6. $\sum \frac{\arctan n}{n^2}$
7. $\sum \frac{5}{\sqrt{n}}$
8. $\sum 5n^{-\sqrt{3}}$

Complete the following series for convergence or divergence. You may use any tests you want. If it is geometric, find the sum.

9. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n}+5}$
10. $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$
11. $\sum_{n=1}^{\infty} \frac{3n^2}{n!}$
12. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$
13. $\sum_{n=1}^{\infty} \frac{3^{n+1}+4^n}{2^{3n}}$
14. $\sum_{n=1}^{\infty} \left(\frac{3^n}{2n}\right)^n$
15. $\sum_{n=2}^{\infty} ne^{-n^2}$
16. $\sum \frac{n}{n^2+1}$

PREVIEW

AP Calculus BC Target Practice
Advanced Integration

Name:

Use the integration rules to evaluate each integral.

1. $\int \frac{x}{(x+5)(x-3)}\,dx$
2. $\int x^2\cos(2x)\,dx$
3. $\int \arcsin(x)\,dx$
4. $\int \frac{\ln(\ln x)}{x}\,dx$
5. $\int \frac{x^2}{e^{3x}}\,dx$
7. $\int \arctan(x)\,dx$
8. $\int x^2\sin(4x)\,dx$
9. $\int \frac{1}{x^2+6x+13}\,dx$
10. $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$
11. $\int_{\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$
12. $\int x\ln(x)\,dx$
13. $\int_2^3 \frac{}{\sqrt{x^2-}}\,dx$
15. $\int_0^2 x\sqrt{5-x^2}\,dx$
16. $\int_0^2 3x\,e^{x^2+1}\,dx$

AP Calculus BC Target Practice
Arc Length and Improper Integrals

Name:

Write an expression for the definite integral that could represent the arc length on the given interval.

1. $y=\sqrt{x}$ over the interval $[0,3]$
2. $y=\sin(2x)$ over one period of the curve.

Find the arc length of the graph on the given interval.

3. $f(x)=\frac{2(x-8)^{3/2}}{3}$ on the interval $[8,16]$
4. $g(x)=\frac{}{3}(x^2-)^{3/2}$ on the interval $[1,4]$

$\ln(\cos x)$ on $\left[0,\frac{\pi}{4}\right]$

Evaluate the following improper integrals.

7. $\int_{-2}^{\infty} \frac{1}{x+1}\,dx$
8. $\int_2^{\infty} xe^{-}\,dx$
9. $\int^{\infty} \frac{1}{x^2+4}\,dx$
10. $\int_e^{} \frac{1}{x(\ln x)^2}\,dx$
11. $\int_0^e \frac{1}{x}\,dx$
12. $\int_1^2 \frac{dx}{\sqrt[3]{x-1}}$
13. $\int_0^{\infty} e^{-x}\,dx$
14. $\int_2^4 \frac{1}{(x-3)^2}\,dx$
15. $\int_2^4 \frac{1}{(x-3)^{2/3}}\,dx$
16. $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

PREVIEW

AP Calculus BC Target Practice

Parametric and Vector-Valued Functions

Name:

Determine the interval(s) on which the vector-valued function is continuous.

1. $r(t) = t\mathbf{i} + \frac{1}{t}\mathbf{j}$

2. $r(t) = \sqrt{t}\,\mathbf{i} + \sqrt{t-3}\,\mathbf{j}$

3. $\boldsymbol{r} = \langle \sin(\boldsymbol{t}), -\cos(\boldsymbol{t}) \rangle$

4. $r(t) = \sin^{-1}(t)\,\mathbf{i}$

For each of the following, find **(a)** $\mathbf{v}'(t)$, **(b)** $\mathbf{v}''(t)$ and **(c)** $D_t[\mathbf{u}(t) \cdot \mathbf{v}(t)]$

5. $\mathbf{v}(t) = t\mathbf{i} + t^2\mathbf{j}$
$\mathbf{u}(t) = 4t\mathbf{i} - t^3\mathbf{j}$

6. $\mathbf{v}(t) = \sin(t)\mathbf{i} +$
$\mathbf{u}(t) = \frac{3}{t}\mathbf{i} - e^t\mathbf{j}$

For each of the following, evaluate the integral.

7. $\int [(2t-3)\mathbf{i} - 6t^2\mathbf{j}]\,dt$

8. $\int \left[\cos(t)\mathbf{i} - \frac{1}{1+t^2}\mathbf{j}\right]\,dt$

For each of the following, find $r(t)$ using the given

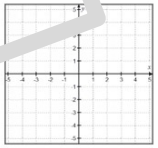
9. $r'(t) = (6e^{2t})\mathbf{i} + (3e^t)\mathbf{j}$
$r(0) = 9\mathbf{i} - \mathbf{j}$

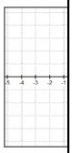
10. $r''(t) = 6t\mathbf{i} + \cos(t)\mathbf{j}$
$r'(0) =$ and $r(0) = 5\mathbf{j}$

The position vector $r(t)$ describes the p in the xy-plane of a oving object. Sk the path and the velocity d acceleration ectors at the given int.

11. $r(t) = 2t\mathbf{i} +$ $t^2)\mathbf{j}$ at $t = 1$

12. $r(t) =$ $\mathbf{i} + t\mathbf{j}$ at $t = 1$





For each of the following, evaluate the partic motion.

13. The positio particle the xy-plane is en by the parametric equations below. For hat alue of t is the particle at r ?
$t) = t^3 - 6t^2$ and $y(t) = t$ $- 3t^2 - 24t$

14. A p ves along a curve so that $\frac{dx}{dt} =$ $\sin t$ and $\frac{dy}{dt} = t^2 + 2\cos(t)$ at $t = 1$, the particle at the oi $(5, -8)$. Find the p of the particle at the point when $t = 2$.

15. A particle moves along a curve so that $\frac{dx}{dy} = \sin(e^{1.035})$ and $\frac{dy}{dt} = \sqrt{t^2+7}$. Find the total distance traveled by the particle over the time interval $1 \le t \le 3$. Round your answer to three decimal places.

16. A particle moves in the xy-plane so that $\frac{dx}{dt} = \sin(t^2)$ and $\frac{dy}{dt} = e^{\cos(t)}$. At $t = 4$, the particle is at the position $(-2, 8)$. Write the equation of the tangent line to the curve at the point where $t = 4$.

PREVIEW

AP Calculus BC Target Practice
Power Series and Intervals of Convergence

Name:

For each of the following, find the interval of convergence of the power series. Check for convergence at the endpoints.

1. $\sum_{k=0}^{\infty} \frac{x^k}{k+1}$

2. $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^k}{\sqrt{k}}$

3. $\sum_{k=1}^{\infty} \frac{(2x)^k}{k!}$

For each of the following, write a power series for the function centered at $x = c$. Give the first four nonzero terms and the general term.

5. $f(x) = \frac{3}{x+2}, c = 0$

6. $f(x) = \frac{1}{x}, c = 1$

7. $f(x) = \frac{1}{1-2x}, c = 0$

8. $f(x) = \ln(1-x), c = 0$

AP Calculus BC Target Practice
Maclaurin and Taylor Polynomials

Name:

Definition of Taylor and Maclaurin Series

If a function f has derivatives of all order at $x = c$, then the series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n = f(c) + f'(c)(x-c) + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \cdots$$

Is called the **Taylor series for $f(x)$ at c.**

Also, if $c = 0$, then the series is the **Maclaurin series for f.**

Find the Maclaurin polynomial of degree 4 for the following functions.

1. $f(x) = \cos(\pi x)$

2. $f(x) = e^{-x}$

3. $f(x) = \ln(1-x)$

For each of the following, find the Taylor polynomial of degree n for the given function centered at $x = c$.

4. $f(x) = \frac{1}{x}, n = 4, c = 1$

6. $f(x) = e^x, n = 4, c = 1$

7. $f(x) = x\cos(x), n = 2, c = \pi$

8. $f(x) = \sin(\pi x), n = 4, c = \frac{1}{2}$

PREVIEW