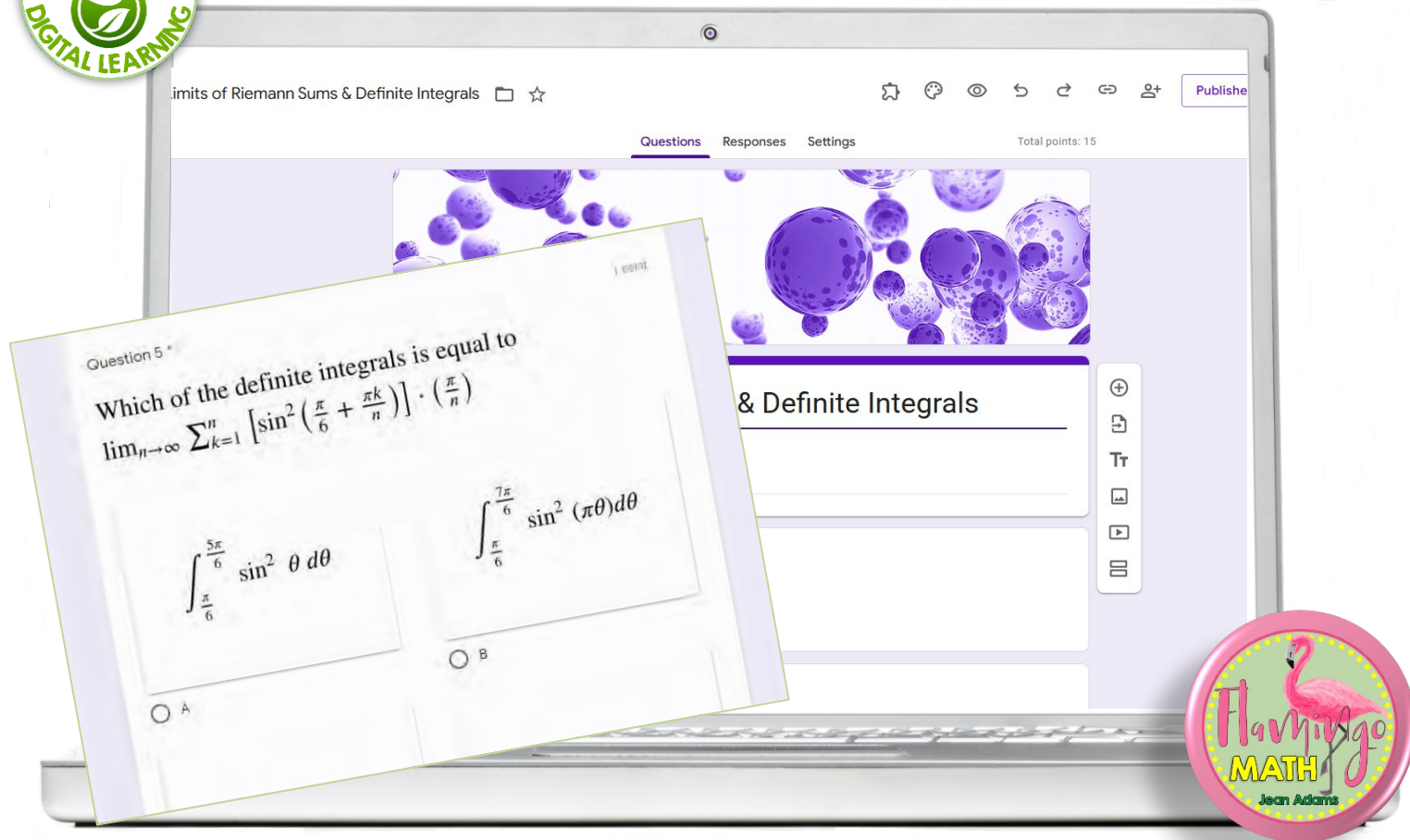
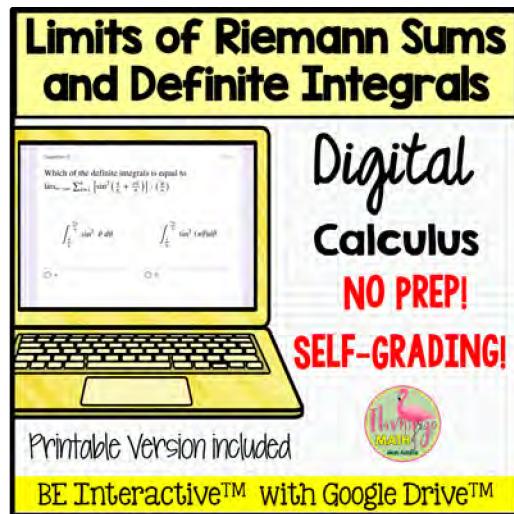


Limits of Riemann Sums and Definite Integrals



Digital and Print



Limits of Riemann Sums and Definite Integrals with Google Forms™

Your AP Calculus students will have 16 self-grading questions related to your unit on Integration – The Accumulation of Change which can be used as an assessment, homework, or a class assignment.

Teachers have the benefit of a paperless, **NO PREP** assignment designed with Google Forms™ or a printable version, if you need this option.

QUESTION STEMS:

1. Write definite integral as a limit of a Riemann Sum.
2. Find definite integral using a Trapezoidal Rule.
3. Use properties of definite integrals to evaluate a sum or difference.
4. Given the limit of a Riemann Sum on an interval $[a, b]$, evaluate the definite integral.
5. Write a definite integral from the limit of a Riemann Sum.
6. Translate the definite integral as a limit of a Riemann Sum, do not evaluate.
7. Given function values in a table, find the definite integral using right Riemann Sums.
8. Find a graph that will underestimate on trapezoidal sums and overestimate on right Riemann sums.
9. Use properties of definite integrals to resolve the integral for a transformation of the integral.
10. Given the limit of a right Riemann Sum, evaluate.
11. Write a definite integral given the expansion of a summation.
12. Find the velocity of an object given initial values and the acceleration in a table by using left-hand sums.
13. Given a piecewise graph and an integral, evaluate on $[a, b]$.
14. Given a table of rate values for a process, use a mid-point sum to evaluate total water flow.
15. Write a definite integral given a limit of a Riemann Sum.
16. Given the limit of a Riemann sum, write the integral.

NOTE: You and your students must have a free Google account in order to access and interact with the activity. It is not necessary to use Google Classroom to use this resource.

GOOGLE FORMS™ are editable. This offers you flexibility to add, delete, or edit questions. You can also adjust the point value for each question to meet your needs.





Limits of Riemann Sums & Integrals

* Required

Name *

Your answer

Class Period *

- ☐ 1st Period
- ☐ 2nd Period
- ☐ 3rd Period
- ☐ 4th Period

Question 5 *

1 point

The value of the definite integrals is equal to

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\sin^2 \left(\frac{\pi}{6} + \frac{\pi k}{n} \right) \right] \cdot \left(\frac{\pi}{n} \right)$$

Question 1 *

1 point

Write the definite integral as a limit of a Riemann Sum

$$\int_1^5 x^3 dx$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{4k}{n} \right)^3 \left(\frac{4}{n} \right)$$

☐ A

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{4k}{n} \right)^3 \left(\frac{4}{n} \right)$$

☐ B

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{k}{n} \right)^3 \left(\frac{4}{n} \right)$$

☐ C

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{k}{n} \right)^3 \left(\frac{1}{n} \right)$$

☐ D

Editable

Question 7 *

1 point

x	2	5	9	15
$f(x)$	10	22	36	28

The function f is continuous on the closed interval $[2, 15]$

and has values as shown in the table.

On the intervals $[2, 5]$, $[5, 9]$, and $[9, 15]$ what is the value of $\int_2^{15} f(x) dx$ found by using a

322

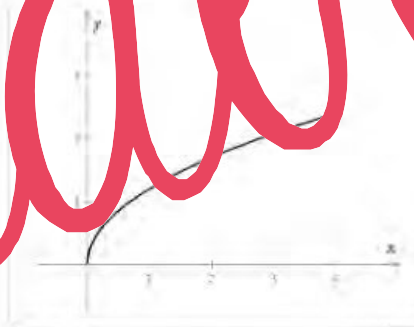
☐ A

378

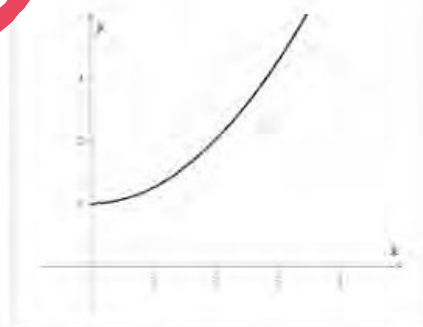
If a trapezoidal sum underapproximates $\int_0^4 f(x) dx$ and a right Riemann Sum overapproximates $\int_0^4 f(x) dx$ which of the following could be the graph of $y = f(x)$?



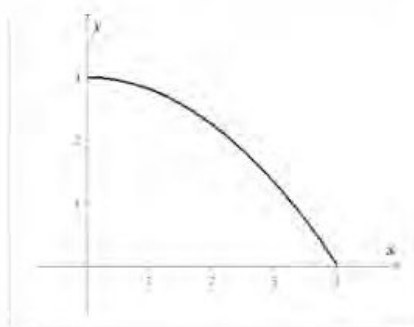
☐ A



☐ B



☐ C



☐ D

Question 10 *

1 point

Find the following limit

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2\sqrt{2+5k}} \right) \cdot \left(\frac{5}{n} \right)$$

$$\frac{1}{2}(\sqrt{7} - \sqrt{2})$$

☐ B

$$\sqrt{7} - \sqrt{2}$$

☐ D

1. Write the definite integral as a limit of a (right) Riemann Sum. $\int_1^5 x^3 dx$

[A.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{4k}{n}\right)^3 \left(\frac{1}{n}\right)$

[B.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{4k}{n}\right)^3 \left(\frac{4}{n}\right)$

[C.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{k}{n}\right)^3 \left(\frac{4}{n}\right)$

[D.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{k}{n}\right)^3 \left(\frac{1}{n}\right)$

2. If three equal subdivisions of $[0, 3]$ are used, what is the Trapezoidal Rule approximation

$\int_0^3 (12 - x^2) dx$

[A.] 26.5

[B.] 27

[C.] 34

[D.] 53

3. If $\int_2^{-6} f(x) dx = -8$ and $\int_2^5 f(x) dx = 16$ what is $\int_{-6}^5 f(x) dx$?

[A.] 8

[B.] -8

[C.] -24

[D.] 24

t (sec)	0	3	6	9
$a(t)$ (ft/sec ²)	4	3	6	2

The data for the acceleration $a(t)$ of a scooter from 0 to 9 seconds are given in the table above. If the velocity at $t = 0$ is 0 feet per second, the approximate value of the velocity at $t = 9$ computed using a left Riemann sum with three subintervals of equal length, is

[A.] $42 \frac{ft}{sec}$

[B.] $49 \frac{ft}{sec}$

[C.] $48 \frac{ft}{sec}$

[D.] 5

13. Use the graph of g shown below to evaluate the right Riemann sum

$\int_0^8 g(x) dx$

[A.] $\frac{9\pi - 30}{4}$

[B.] $\frac{9\pi - 15}{4}$

[C.] $\frac{9\pi - 15}{2}$

[D.] $\frac{9\pi}{4} - 8$

14. The table below shows the rate of water flow $w(t)$ through a pipe, in gallons per hour from $0 \leq t \leq 12$ hours. Estimate the total amount of water that flowed through the pipe, in gallons, for the 12-hour period by using a mid-point Riemann Sum with three sub-intervals.

t (hours)	0	2	4	5	7	9	12
$w(t)$ (gals/hr)	0	6	10	12	14	11	4

[A.] 186 gallons

[B.] 348 gallons

[C.] 115 gallons

[D.] 139 gallons

5. Which of the definite integrals is equal to $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\sin^2 \left(\frac{\pi}{6} + \frac{\pi k}{n} \right) \right] \cdot \left(\frac{\pi}{n} \right)$

[A.] $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin^2 \theta d\theta$

[B.] $\int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \sin^2(\pi\theta) d\theta$

[C.] $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin^2(\pi\theta) d\theta$

[D.] $\int_{\frac{\pi}{6}}^{\frac{7\pi}{6}} \sin^2 \theta d\theta$

6. Translate the definite integral into a Riemann sum. Do not evaluate. $\int_2^5 (3x^2 + 1) dx$

[A.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[3 \left(2 + \frac{3k}{n} \right)^2 + 1 \right] \cdot \left(\frac{3}{n} \right)$

[B.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[3 \left(2 + \frac{3k}{n} \right)^2 + 1 \right] \cdot \left(\frac{3}{n} \right)$

[C.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[3 \left(2 + \frac{3k}{n} \right)^2 + 1 \right] \cdot \left(\frac{3}{n} \right)$

[D.] $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[3 \left(2 + \frac{3k}{n} \right)^2 + 1 \right] \cdot \left(\frac{3}{n} \right)$

15. Write the definite integral that is approximated by the limit

$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[2 + \sqrt{2 + \frac{3k}{n}} \right] \cdot \left(\frac{3}{n} \right)$

[A.] $\int_0^4 \sqrt{2+x} dx$

[C.] $\int_1^4 \sqrt{2+x} dx$

16. Which of the definite integrals is equal to $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\ln \left(e + \frac{ek}{n} \right) \right] \cdot \left(\frac{e}{n} \right)$

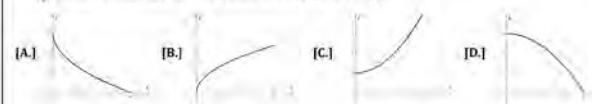
[A.] $\int_0^{2e} \ln x dx$

[B.] $\int_e^{2e} \ln x dx$

[C.] $\int_e^{2e} \ln(e+x) dx$

[D.] $\int_0^e \ln(ex) dx$

8. If a trapezoidal sum underapproximates $\int_a^b f(x) dx$ and a right Riemann Sum overapproximates $\int_a^b f(x) dx$ which of the following could be the graph of $y = f(x)$?



9. Find $\int_1^6 f(x) dx$ given $\int_1^6 [2f(x) + 3] dx = 25$

[A.] 11

[B.] 14

[C.] 5

[D.] 12.5

10. Find the full limit: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{2 + \frac{5k}{n}}}$

[A.] $\sqrt{5} - \frac{1}{\sqrt{2}}$

[B.] $\frac{1}{2}(\sqrt{5} - \sqrt{2})$

[C.] $\frac{1}{2}(\sqrt{5} - \sqrt{2})$

[D.] $\frac{1}{2}(\sqrt{5} - \sqrt{2})$

11. $\frac{\pi}{6} \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) + \dots + \cos\left(\frac{6\pi}{6}\right)$

The expression above is a right Riemann Sum approximation for which of the following?

[A.] $\frac{1}{6} \int_0^{\pi} \cos x dx$

[B.] $\int_0^{\pi} \cos x dx$

[C.] $\frac{1}{6} \int_0^{\pi} \cos\left(\frac{x}{6}\right) dx$

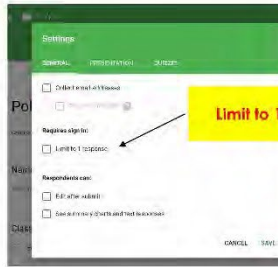
[D.] $\int_0^{\pi} \cos\left(\frac{x}{6}\right) dx$

SELF-GRADING

First, The Basics:

- 1.) Have access to the Internet.
- 2.) Have a Google account – this is FREE and easy to create.
- 3.) Every student who uses the activity needs their own Google Account and the Google Forms App.
- 4.) Students can submit their finished school is one-to-one. Using Google is not a requirement.
- 5.) If you want to make a hard copy

Click the Gear and Check S



Changing Point Values:

Every question is set at a value of 1 point each.

16 questions show total points: Total points: 16

To alter the point values:

- 1.) Click on any question.
- 2.) Click ANSWER KEY (located at the bottom of the question).
- 3.) Click on arrow up/down to change the point value.

Self-Grading

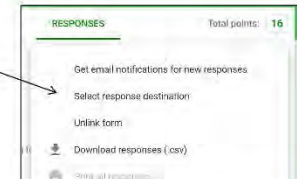
From your teacher copy you can see the GOOGLE SHEET to track student data.

On the RESPONSES TAB:

Click the three vertical dots (snowman) to begin the set-up for creating a spreadsheet.

Self-Grading Set-Up & Spreadsheet:

Click **SELECT RESPONSE DESTINATION**. Then create a new spreadsheet. This will create the spreadsheet and will save by default to your GOOGLE DRIVE Folder

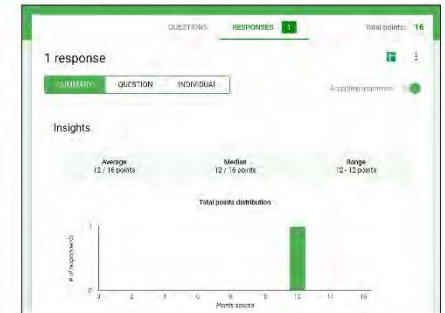


To view the student responses on the spreadsheet click here:

Viewing Student Responses:

Teachers can check responses through the **RESPONSE Tab** using a bar graph **SUMMARY**, a **QUESTION** report or an **INDIVIDUAL** student report.

If you choose to create a GOOGLE SHEET, the data will also be collected for students in the **SPREADSHEET**, which will allow sorting and an overview of all responses as shown below.



Polynomial and Rational Functions (Responses)				
File Edit View Insert Format Data Tools Form Add-ons Help				
Working...				
Timestamp	A	B	C	D
1	Timestamp	Score	Name	Class Period
2	10/26/2019 7:54:10	11 / 16	Jean Adams	1st period
3				
4				

Low Prep



Step-by-Step Instructions

The Basics for using this Google Forms file, you must:

- 1.) Have access to the Internet.
- 2.) Have a Google account – this is FREE and easy to create.
- 3.) Every student who uses the activity needs their own Google Account and the Google Forms App.
- 4.) For those who wish to make a hard copy of the finished assignment, you may want to have access to a printer. This is optional. Students can submit their finished activity through your learning management system, if your school is one-to-one.

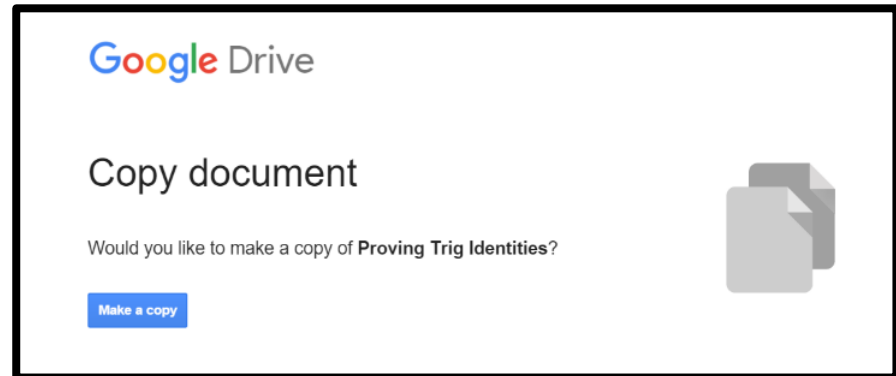
5.) Here is a link to the product:

<https://tinyurl.com/yc4r3m8h>

6.) Upon clicking link, a screen will prompt you to make a copy of the file.

Click “Make a copy.”

7.) When you give your students a link, be sure that they are instructed to make a copy too! By doing this, each user will have their own activity on which to work – they won’t accidentally be working on someone else’s document.



NOTE: You and your students must have a free Google account in order to access and interact with the activity. It is not necessary to use Google Classroom to use this resource.

GOOGLE FORMS™ are editable. This offers you flexibility to add, delete, or edit questions. You can also adjust the point value for each question to meet your needs

[Download Google Form Here](#)

First, The Basics:

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- 2.) Have a Google account – this is FREE and easy to create.
- 3.) Every student who uses the activity needs their own Google Account and the Google Forms App.
- 4.) Students can submit their finished assignment through your learning management system, if your school is one-to-one. Using Google Classroom is not a requirement.
- 5.) If you want to make a hard copy of the assignment, you need access to a printer. This is optional.

Click the Gear and Check Settings:



Settings

GENERAL PRESENTATION QUIZZES

☐ Collect email addresses

☐ Response receipts ?

Requires sign in:

☐ Limit to 1 response

Respondents can:

☐ Edit after submit

☐ See summary charts and text responses

CANCEL SAVE

Settings

GENERAL PRESENTATION QUIZZES

☒ Make this a quiz
Assign point values to questions and allow auto-grading.

Quiz options

Release grade:

☒ Immediately after each submission

☐ Later, after manual review
Turns on email collection

Respondent can see:

☒ Missed questions ?

☒ Correct answers ?

☒ Point values ?

CANCEL SAVE

Changing Point Values:

Every question is set at a value of 1 point each.

20 questions show total points:

Total points:
20

To alter the point values:

- 1.) Click on any question.
- 2.) Click ANSWER KEY (located at the bottom of the question)
- 3.) Click on arrow up/down to change point value for each question.

 **ANSWER KEY** (1 point)

1  **points**

5 **points**

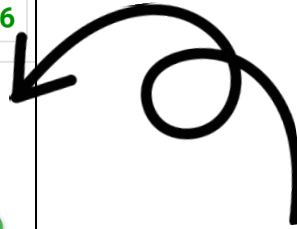
Self-Grading Set-Up & Spreadsheet:

From your teacher copy you can see responses in several different ways. You can also create a GOOGLE SHEET to track student data.

On the **RESPONSES TAB**:

Click the three vertical dots (snowman) to begin the set-up for creating a spreadsheet.

QUESTIONS	RESPONSES	Total points: 16
		
	Accepting responses	
Waiting for responses		


Click here

Self-Grading Set-Up & Spreadsheet:

Click **SELECT RESPONSE DESTINATION**,
Then create a new spreadsheet.
This will create the spreadsheet and will
save by default to your GOOGLE DRIVE
Folder

Select response destination ×

☒ Create a new spreadsheet Polynomial and Rational F... [Learn More](#)

☐ Select existing spreadsheet

CANCEL CREATE

RESPONSES Total points: 16

[Get email notifications for new responses](#)

[Select response destination](#)

[Unlink form](#)

 [Download responses \(.csv\)](#)

 [Print all responses](#)

To view the student responses on
the spreadsheet click here:

QUESTIONS **RESPONSES** Total points: 16

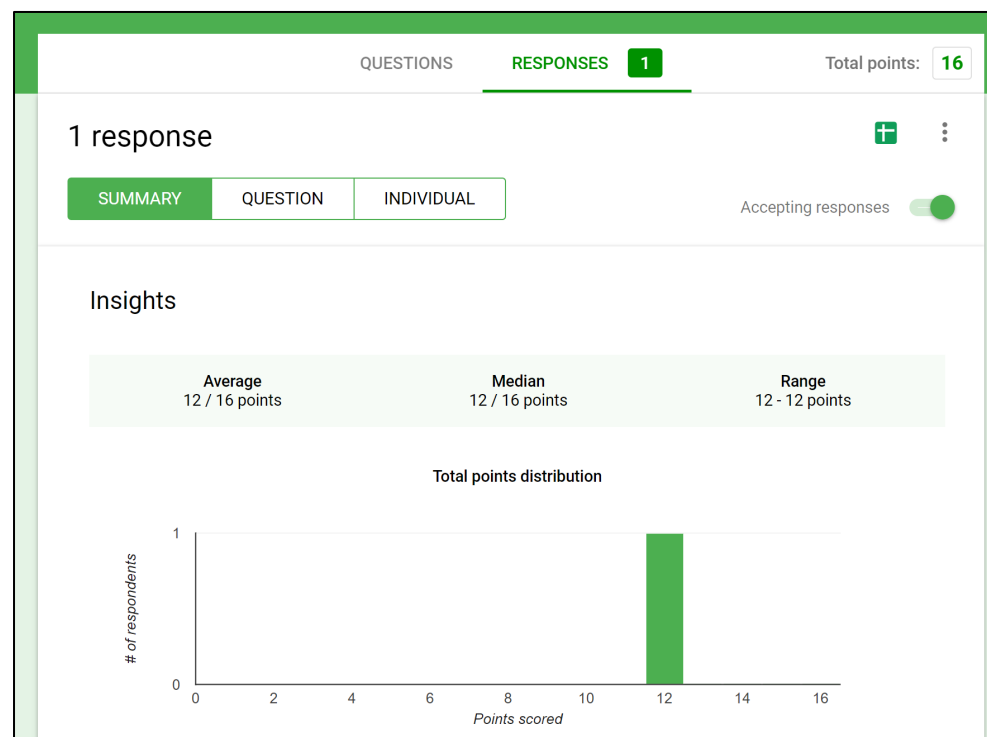
spreadsheet Accepting responses ☒

Waiting for responses

Viewing Student Responses:

Teachers can check responses through the **RESPONSE Tab** using a bar graph **SUMMARY**, a **QUESTION** report or an **INDIVIDUAL** student report.

If you choose to create a GOOGLE SHEET, the data will also be collected for students in the **SPREADSHEET**, which will allow sorting and an overview of all responses as shown below.



The screenshot shows a Google Sheet titled 'Applications of Trigonometry (Responses)'. The sheet has a menu bar with 'File', 'Edit', 'View', 'Insert', 'Format', 'Data', 'Tools', 'Extensions', and 'Help'. Below the menu bar is a toolbar with various icons and settings. The sheet contains a table with the following columns: 'Timestamp', 'Score', 'Student Name', 'What class period is this?', and 'Question #1'. The table has 7 rows, with the first row containing headers and the subsequent rows being empty.

	Timestamp	Score	Student Name	What class period is this?	Question #1
1					
2					
3					
4					
5					
6					
7					

SAVE A COPY OF YOUR COMPLETED ASSIGNMENT!

**THIS IS JUST AS A PRECAUTION THAT SOMETHING MIGHT
GO WRONG!**

**YOU CAN ALSO PRINT A COPY OF THE ASSIGNMENT IN THE
EVENT THAT YOUR INTERNET CONNECTION IS DOWN ON
THE DAY YOU WANT TO ASSIGN THE ACTIVITY!**



Sending the Form to Students:

When you are ready to assign to your students, you can send the form through **email** or by posting a **link to the form** on your school's learning management system for students to access. Let's get started:



Click the SEND button:

Send form

☐ Collect email addresses

Send via

Email

To

Subject

Polynomial and Rational Functions

Message

I've invited you to fill out a form:

☐ Include form in email

[Add collaborators](#)

CANCEL

SEND

SENDING BY EMAIL

1. Choose send via email.
2. Type the student email address.
3. Write a brief message.
4. Decide if you want to include the form in the email.
5. Send

Sending the Form to Students:

If you want to share the form through your learning management system.



Click the **SEND** button:

Send form

☐ Collect email addresses

Send via     

Link
<https://forms.gle/L2sFiDqYt6WF1LP18>

☒ Shorten URL

CANCEL COPY

POSTING THE LINK

1. Open the Google Form.
2. In the top right, click **SEND**.
3. At the top of the window, click Link.
4. Click shorten URL
5. Click Copy.
6. Paste this link into your LMS or preferred method of notifying students.

Answer Key:

If you need the multiple-choice responses. . .

Limits of Riemann Sums and Definite Integrals Key	
1. B	11. B
2. A	12. C
3. D	13. A
4. C	14. C
5. D	15. D
6. A	16. B
7. C	
8. B	
9. C	
10. D	



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- Make copies for, email to, or share with others.
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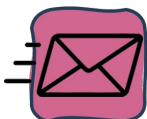
Meet the author:



My goal is to help you bring balance into your teacher life by providing relevant content, engaging activities, and comprehensive assessments for your high-stakes math course.

Jean Adams

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Credits:

