AP Calculus AB/BC #1 Key 2025

1A) $\frac{1}{4}\int_0^4 C(t)\,dt = \underline{2.778}$

1B) $C'(t) = \frac{C(4)-C(0)}{4-0}$

$\underline{t = 2.154}$

1C) $\lim_{t\to\infty}\left[\frac{38}{25+t^2}\right] = 0$

1D) $A(t) = C(t) - \int_4^t 0.1\ln(x)\,dx$

$A'(t) = \frac{38}{25+t^2} - 0.1\ln(t)$

$A'(t) = 0$

$t = 11.4416998$

By candidates test

t	A(t)
4	5.128
11.4416	7.317
36	1.743

$\therefore$ Maximum occurs at either a critical point or an endpoint. At $t = 11.442$ weeks $A(t)$ reaches a maximum value.

AP Calculus BC #2 Key 2025

2A) $r(\theta) = 2\sin^2\theta \qquad 0 \le \theta \le \pi$

$$\frac{dr}{d\theta} = 4\sin\theta\cos\theta$$

$$\left.\frac{dr}{d\theta}\right|_{\theta=1.3} = 4\sin(1.3)\cos(1.3) = \underline{\underline{1.03100}}$$

2B) $2\sin^2\theta = \frac{1}{2}$ intersections on curve at

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\text{Area} = \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[(2\sin^2\theta)^2 - \left(\tfrac{1}{2}\right)^2\right]d\theta$$

$$\text{Area} = \frac{20\pi + 21\sqrt{3}}{48} \approx \underline{\underline{2.06676}}$$

2C) $r(\theta) = 2\sin^2\theta$

$x(\theta) = 2\sin^2\theta\cos\theta$

$$\frac{dx}{d\theta} = 4\sin\theta\cos^2\theta - 2\sin^3\theta$$

$$\frac{dx}{d\theta} = 0 \text{ at } x = 0.9553166\ldots$$

$|x|$ is at max. and farthest from y-axis when $x(\theta)$ has max or min.

θ	$x(\theta)$
0	0
0.9553	0.769800
$\pi/2$	0

$\therefore$ The point on the curve of C that is farthest from the y-axis occurs when $\theta = 0.9553$.

2D) $\frac{dr}{d\theta} = 4\sin\theta\cos\theta\,\frac{d\theta}{dt}$ at $\theta = 1.3$

$$\frac{dr}{d\theta} = 4\sin(1.3)\cos(1.3)(15) \approx \underline{15.4650}$$

AP Calculus AB/BC #3 2025

3A) $R'(1) \approx \frac{R(2)-R(0)}{2-0} = \frac{100-90}{2-0} = 5 \ \frac{\text{words/min}}{\text{min}}$

3B) Since it is given that R is differentiable, we also know that R is continuous. $R(8)=150$ and $R(10)=162$, so $150<155<162$, there must be a time c on the interval $0<c<10$ where $R(c)=155$ this specifically on $8<c<10$.

3C) $\int_0^{10} R(t)\,dt = \frac{1}{2}(2)[90+100] + \frac{1}{2}(6)[100+150] + \frac{1}{2}(2)[150+162]$

$= \underline{1252 \text{ words}}$

3D) $\int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100\right)dt$

$= -\frac{t^3}{10} + 4t^2 + 100t \Big|_0^{10}$

$= 1300 \text{ words}$

$\therefore$ The teacher has read 1300 words in $0 \leq t \leq 10$ minutes.

AP Calculus AB/BC #4 2025

4A) $g'(x) = f(x)$

$g'(8) = f(8) = 1$

4B) On $-6 < x < 12$ g has a point of inflection at $x = -3$ and $x = 6$ because $g'(x) = f(x)$ changes from decreasing to increasing. Also, g has a point of inflection at $x = 3$ because $g'(x) = f(x)$ changes from increasing to decreasing.

4C) $g(12) = \int_6^{12} f(t)\,dt = 9$

$g(0) = \int_6^0 f(t)\,dt = -\int_0^6 f(t)\,dt = -\frac{9\pi}{2}$

4D) $g'(x) = 0$ at $x = -6$, $x = 0$, and $x = 6$

$g(-6) = \int_6^{-6} f(t)\,dt = -\int_{-6}^{6} f(t)\,dt = 0$

$g(0) = -\frac{9\pi}{2}$

$g(6) = \int_6^6 f(t)\,dt = 0$

$g(12) = 9$

$\therefore$ On $-6 \leq x \leq 12$, the absolute minimum value of g occurs at $x = 0$.

AP Calculus BC #5 2025

5A) $\frac{d^2y}{dx^2} = (3-x)(2y)\frac{dy}{dx} - xy^2$

$= (3-x)(2y)(3-x)(y^2) - xy^2$

$\left.\frac{d^2y}{dx^2}\right|_{(1,-1)} = (2)(-2)(2)(1) - 1(1) = \underline{\underline{-9}}$

5B) $T(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2}$ $f(1) = -1$

$T(x) = -1 + 2(x-1) - \frac{9}{2!}(x-1)^2$

5C) Error $\leq \left| \frac{\max(f'''(x))(1.1-1.0)^3}{3!} \right|$ for $1 \leq x \leq 1.1$

Error $\leq \frac{60}{3!}(1.1-1.0)^3 \leq \frac{1}{100} = \underline{\underline{0.01}}$

5D) $f(1.2) \approx f'(1)[1.2-1] - 1 = 2(0.2) - 1 = -0.6$

$\left.\frac{dy}{dx}\right|_{(1.2,-0.6)} = (3-1.2)(-0.6)^2 = 0.648$

$f(1.4) \approx 0.648(0.2) - 0.6 \approx \underline{\underline{-0.4704}}$

AP Calculus BC #6 2025

6A) $\lim_{n\to\infty}\left[\frac{a_{n+1}}{a_n}\right] = \lim_{n\to\infty}\left|\frac{(x-4)^{\cancel{n+2}\,1}}{(n+2)\,3^{\cancel{n+1}}}\cdot\frac{(n+1)\cancel{3^n}}{\cancel{(x-4)^{n+1}}}\right|$

$= \lim_{n\to\infty}\left|\frac{(x-4)(n+1)}{3\,(n+2)}\right| \Rightarrow \left|\frac{x-4}{3}\right|\cdot\lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| \xrightarrow{} 1$

$\Rightarrow \left|\frac{x-4}{3}\right|$

$-1 < \frac{x-4}{3} < 1$

$-3 < x-4 < 3$

$1 < x < 7$ interval of convergence

Check endpoints, when $x=1$;

$\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{(n+1)3^n} \Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3)^{n+1}}{(n+1)\,3^n}$

$\lim_{n\to\infty}\frac{3^{n+1}}{(n+1)(3)^n} = 0$ converges by Alternating Series Test

When $x=7$

$\sum_{n=1}^{\infty}\frac{(3)^{n+1}}{(n+1)3^n} \Rightarrow \sum_{n=1}^{\infty}\frac{3}{(n+1)}$ series diverges

by Limit Comparison Test $\frac{3}{n+1} > \frac{1}{n}$ for $n\geq 1$

also, $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.

$\underline{\underline{1 \leq x < 7}}$

6B) $f'(x) = \frac{2(x-4)}{2\cdot 3} + \frac{3(x-4)^2}{3\cdot 3^2} + \frac{4(x-4)^3}{4\cdot 3^3} + \ldots \frac{(n+1)(x-4)^n}{(n+1)\,3^n}$

$f'(x) = \frac{x-4}{3} + \frac{(x-4)^2}{3^2} + \frac{(x-4)^3}{3^3} + \ldots \frac{(x-4)^n}{3^n} + \ldots$

6C) $f'(x) = \sum_{n=1}^{\infty} \left(\frac{x-4}{3}\right)^n$ geometric: $r = \frac{x-4}{3}$

inf. geom. seq. $= \frac{a_1}{1-r}$

$$= \frac{\frac{x-4}{3}}{1-\left(\frac{x-4}{3}\right)} = \frac{\frac{x-4}{\cancel{3}}}{\frac{3-x+4}{\cancel{3}}} = \frac{x-4}{7-x}$$

6D) The interval of convergence is $-1 \leq x < 7$ for both f and f'.
Note that 8 is not in this interval, and $f'(x)$ does not converge to $\frac{x-4}{7-x}$ at $x = 8$

Also, $f'(8) = \sum_{n=1}^{\infty} \left(\frac{8-4}{3}\right)^n = \sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

geometric $\frac{4}{3} > 1$, $f'(x)$ diverges.