

AP Calculus AB/BC #1 Key

2025

1A) $\frac{1}{4} \int_0^4 C(t) dt \approx \underline{\underline{2.7782 \text{ acres}}}$

1B) $C'(t) = \frac{C(4) - C(0)}{4 - 0}$
 $\underline{\underline{t = 2.1542 \text{ weeks}}}$

1C) $\lim_{t \rightarrow \infty} \left[\frac{38}{25 + t^2} \right] = 0$

1D) $A(t) = C(t) - \int_4^t 0.1 \ln(x) dx$
 $A'(t) = \frac{38}{25 + t^2} - 0.1 \ln(t)$

$$A'(t) = 0$$

$$t = 11.4416998$$

By candidates test

t	$A(t)$
4	5.128
* 11.4416	7.3169
36	1.743

$\therefore$ Maximum occurs at either a critical point or endpoint. At $t = 11.442$ weeks is time $A(t)$ reaches a maximum value.

AP Calculus AB #2

2025

$$2A) \quad A_R = \int_0^3 [g(x) - f(x)] dx$$

$$\underline{\underline{A_R = 5.136619 \dots}}$$

$$2B) \quad V = \int_0^3 [x[g(x) - f(x)]] dx$$

$$\underline{\underline{V = 7.704929 \dots}}$$

$$2C) \quad V = \pi \left[\int_0^3 (g(x) + 2)^2 - (f(x) + 2)^2 \right] dx$$

$$2D) \quad f'(x) = 2x - 2$$

$$g'(x) = 1 + \pi \cos(\pi x)$$

$$f'(x) \stackrel{?}{=} g'(x)$$

$$x = 0.6758, 1.5, 2.32418$$

$$\therefore \text{On } 0 < x < 1 \quad f'(x) = g'(x) \text{ at } \underline{\underline{x = 0.6758}}$$

AP Calculus AB/BC # 3

2025

$$3A) \quad R'(1) = \frac{R(2) - R(0)}{2 - 0} = \frac{100 - 90}{2 - 0} = 5 \frac{\text{words}}{\text{min}}$$

3B) Yes, there is a value for $0 < c < 10$ where $R(c) = 155$, since $R(t)$ is continuous and differentiable, $f(8) = 150$ and $f(10) = 162$ therefore $f(c) = 155$ on $8 < c < 10$.

$$\begin{aligned} 3C) \quad \int_0^{10} R(t) dt &= \left[\frac{(2-0)}{2} (90+100) + \frac{(8-2)}{2} (100+150) + \frac{(10-8)}{2} (150+162) \right] \\ &= 190 + 3(250) + 312 \\ &= 190 + 750 + 312 \\ &= 1252 \end{aligned}$$

$$\begin{aligned} 3D) \quad \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100 \right) dt &= \left[-\frac{x^3}{10} + 4x^2 + 100x \right]_0^{10} \\ &= -100 + 400 + 1000 \\ &= 1300 \text{ words} \end{aligned}$$

$\therefore$ The teacher has read 1300 words in $0 \leq t \leq 10$ minutes

4A) $g'(x) = f(x)$
 $g'(8) = f(8) = 1$

4B) On $-6 < x < 12$, the graph of g has a point of inflection where the graph of f , the derivative of g , changes from decreasing to increasing or vice versa. Therefore, the graph of g will have points of inflection at $x = -3$, $x = 3$ and $x = 6$.

4C) $g(12) = \int_6^{12} f(t) dt$ $g(0) = -\int_0^6 f(t) dt$
 $g(12) = \frac{1}{2}(6)(3)$ $g(0) = -\left[\frac{\pi}{2}(3)^2\right]$
 $g(12) = 9$ $g(0) = -\frac{9\pi}{2}$

4D) On $-6 \leq x \leq 12$ the absolute minimum value which g attains occurs at $x = 0$.

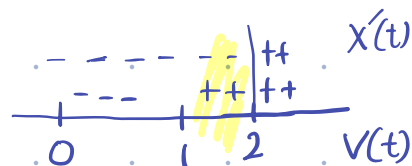
x	$g(x)$
-6	0
0	$-\frac{9\pi}{2}$
6	0
12	9

AP Calculus AB #5

2025

5A) $x_H(t) = e^{t^2 - 4t}$
 $x'_H(t) = (2t - 4)e^{t^2 - 4t}$
 $x'_H(1) = (-2)e^{-3}$

5B) $x'_H(t) = 0$ at $x = 2$
 $v_J(t) = 0$ at $x = -1, 0, 1$



The particles H and J move in opposite directions where their velocities have opposite signs. On interval $1 < t < 2$
 $x'_H(t) < 0$ and $v_J(t) > 0$.

5C) $v_J(2) = 2(2)[2^2 - 1]^3 > 0$ and $v'_J(2) > 0$
 $v_J(2) > 0$ and $v'_J(2) > 0$, so the speed of the particle is increasing since the velocity and acceleration are both positive.

5D) $s_J(t) = s_J(0) + \int_0^t 2t(t^2 - 1)^3 dt$
 $s_J(2) = 7 + \int_{-1}^3 u^3 du$
 $= 7 + \left[\frac{1}{4} u^4 \right]_{-1}^3$
 $= 7 + \frac{1}{4}(80)$

$s_J(0) = 7$
 $u = t^2 - 1$
 $du = 2t dt$
 $t = 0 \quad u = -1$
 $t = 2 \quad u = 3$

$s_J(2) = 27$

AP Calculus AB #6

2025

$$\begin{aligned} \text{6A)} \quad & y^3 - y^2 - y + \frac{1}{4}x^2 = 0 \\ & 3y^2 y' - 2y y' - y' + \frac{1}{2}x = 0 \\ & \frac{dy}{dx} [3y^2 - 2y - 1] = -\frac{1}{2}x \end{aligned}$$

$$\underline{\underline{\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}}}$$

$$\text{6B)} \quad \left. \frac{dy}{dx} \right|_{(2,-1)} = \frac{-2}{2(3+2-1)} = \frac{-1}{4} \quad \text{at } (2,-1)$$

$$\text{Tangent line: } y = -1 - \frac{1}{4}(x-2)$$

$$y(1.6) = -1 - 0.25(1.6-2)$$

$$\underline{\underline{y(1.6) = -0.9}}$$

$$\begin{aligned} \text{6C)} \quad & 3y^2 - 2y - 1 = 0 \\ & (3y+1)(y-1) = 0 \\ & y = -\frac{1}{3}, y = 1 \end{aligned}$$

$\therefore$ For $x > 0$ and $y > 0$
point S has a vertical
tangent line when
 $y = 1$.

$$\begin{aligned} \text{6D)} \quad & 2xy + \ln y = 8 \\ & 2x \frac{dy}{dt} + 2y \frac{dx}{dt} + \frac{1}{y} \frac{dy}{dt} = 0 \end{aligned}$$

$$2(4) \frac{dy}{dt} + 2(3) + \frac{dy}{dt} = 0$$

$$\underline{\underline{\frac{dy}{dt} = -\frac{6}{9}}}$$