# AP Calculus AB/BC #1 Key

2024

A) $C'(5) = \dfrac{C(7) - C(3)}{7-3} = \dfrac{69-85}{7-3} = -4$

$\underline{C'(5) = -4\ C°/\text{min}}$

B) $\int_0^{12} C(t)\,dt = (3-0)(100) + (7-3)(85) + (12-7)(69)$

$= 3(100) + 4(85) + 5(69)$

$= 300 + 340 + 345$

$= 985$

$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of coffee in degrees Celsius as it cools in the cup over time $0 \le t \le 12$ min.

C) $\int_{12}^{20} C'(t)\,dt = C(20) - C(12)$

$-14.6708119444 = C(20) - 55$

$\underline{C(20) \approx 40.32918\ C°}$

D) $C''(t) > 0$ on $12 < t < 20$, so temperature of coffee is changing at an increasing rate on the interval.

# AP Calculus AB/BC #2 Key

A) At $t = 1.42560$ the particle is at rest.
$0 < t < 1.426$ $v(t) > 0$, so the particle is moving to the right.

B) $a(t) = v'(t)$

$$\frac{d}{dt}\left[v(t)\right]_{t=1.5} = -1$$

$a(1.5) < 0$ and $v(1.5) < 0$, so the speed of the particle is increasing at $t = 1.5$ since signs are the same.

C) $x(4) = x(1) + \int_1^4 v(t)\,dt$ $\qquad x(1) = -3$

$= -3 + 0.1971167$

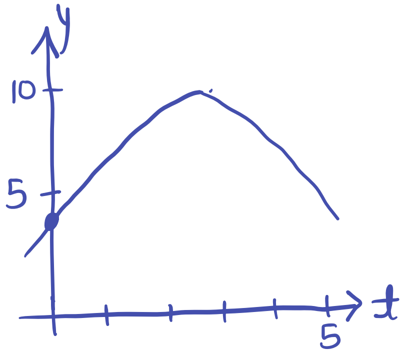
$x(4) = -2.80288$

The position of the particle at time $t = 4$ is $-2.803$.

D) $\int_1^4 |v(t)|\,dt = \underline{0.95813}$

# AP Calculus AB/BC #3 Key

A)




$H(0) = 4$

B) $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\cos\left(\frac{t}{2}\right) = 0$

$t = \pi$

$H'$: $++$ $0$ $--$

$H$: $0$ inc $\pi$ dec $5$

At $t=\pi$ $H'(\pi)=0$ and $H(t)$ changes from increasing to decreasing at $t=\pi$. This critical point is a relative maximum for depth of seawater.

c) $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\int \frac{2}{H-1}\, dH = \int \cos\left(\frac{t}{2}\right) dt$

$2\ln|H-1| = 2\sin\left(\frac{t}{2}\right) + C$

$\ln|H-1| = \sin\left(\frac{t}{2}\right) + C$ $\qquad H(0) = 4$

$\ln|4-1| = \sin 0 + C$

$\ln 3 = C$

$\ln|H-1| = \sin\left(\frac{t}{2}\right) + \ln 3$

$e \qquad\qquad e$

$H - 1 = e^{\sin t/2} \cdot e^{\ln 3}$

$\underline{\underline{H(t) = 3e^{\sin(t/2)} + 1}}$

# AP Calculus AB/BC #4 Key

A) $g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^{0} f(t)\,dt = \underline{\underline{-12}}$

$g(4) = \int_0^4 f(t)\,dt = \frac{1}{2}(4)(2) = \underline{\underline{4}}$

$g(6) = \int_0^4 f(t)\,dt + \int_4^6 f(t)\,dt = 4 + (-1) = \underline{\underline{3}}$

$\underline{\underline{g(-6) = -12;\ g(4) = 4;\ g(6) = 3}}$

B) $g(x)$ has a critical point where $f(x) = g'(x) = 0$. This occurs $\underline{\underline{\text{at } x = 4}}$ $\quad f(4) = g'(4) = 0$

c) $h(x) = \int_{-6}^{x} f'(t)\,dt$

$h(6) = \int_{-6}^{6} f'(t)\,dt = f(6) - f(-6)$

$= -1 - 0.5$

$\underline{h(6) = -1.5}$

$h'(x) = \frac{d}{dx}\left[\int_{-6}^{x} f'(t)\,dt\right] = f'(x)$

$h'(6) = f'(6) = \underline{\underline{-\frac{1}{2}}}$

$h''(6) = f''(6) = \underline{\underline{0}}$

# AP Calculus AB #5 Key

A) $\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{-2(2)}{3+4(4)} = \frac{-4}{19}$

T.L. $y = -\frac{4}{19}(x-2)+4$

$y = -\frac{4}{19}(3-2)+4$

$\underline{\underline{y = -\frac{4}{19}+4}}$

B) If $y=1$ $\quad x^2+3(1)+2(1)^2=48$

$x^2 = 43$

$x = \pm\sqrt{43}$

$(\sqrt{43},1)$
$(-\sqrt{43},1)$

$y'=0 \qquad 0 \neq \frac{-2(\sqrt{43})}{3+4(1)}$

No, $y=1$ is not tangent to curve Point $(\sqrt{43},1)$ or $(-\sqrt{43},1)$ does not satisfy $\frac{dy}{dx}=0$

# #5 Key (cont)

c) $\left.\frac{dy}{dx}\right|_{(\sqrt{48},0)} = \frac{-2\sqrt{48}}{3+4(0)} = \frac{-8\sqrt{3}}{3}$

No, the tangent line at x-axis is not vertical because $\frac{dy}{dx}$ is $-\frac{8\sqrt{3}}{3}$.

D) $y^3 + 2xy = 24$

$3y^2 \frac{dy}{dt} + 2x \frac{dy}{dt} + 2\frac{dx}{dt} y = 0$

At $(4,2)$
$\frac{dy}{dt} = -2$

$3(2)^2(-2) + 2(4)(-2) + 2(2)\frac{dx}{dt} = 0$

$-24 - 16 + 4\frac{dx}{dt} = 0$

$4\frac{dx}{dt} = 40$

$\frac{dx}{dt} = 10$ units/second

# AP Calculus #6 Key

A) $R(x) = \int_0^2 [f(x) - g(x)]\,dx$

B) $S(x) = \int_2^5 \frac{1}{2}[g(x)]^2\,dx$

$$\boxed{\begin{array}{c} A = B \cdot H \\ H = \frac{1}{2}B \\ A = \frac{1}{2}B^2 \end{array}}$$

$= \frac{1}{2}\int_2^5 (x^2 - 2x)^2\,dx$

$= \frac{1}{2}\int_2^5 x^4 - 4x^3 + 4x^2\,dx$

$= \frac{1}{2}\left[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3\right]_2^5$

$= \frac{1}{2}\left[\cancel{625} - \cancel{625} + \frac{4}{3}(125) - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)\right]$

$= \frac{1}{2}\left[\frac{4 \cdot 125}{3} - \frac{32}{5} + 16 - \frac{32}{3}\right]$

$= \frac{414}{5}$

C) $V(x) = \int_2^5 \pi(20)^2 - \pi[20 - g(x)]^2\,dx$