AP Calculus AB #1 Key

A) $\int_1^5 450\sqrt{\sin(0.62t)}\,dt$

B) $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.5369$ vehicles/hour

C) $A'(1) = 148.947$. Since $A'(t) > 0$, the rate at which vehicles arrive at the toll plaza is increasing at 6 A.M.

D) $N'(t) = A(t) - 400$

$A(t) = 400$

$t = 1.469 \quad t = 3.598$

By candidates test

t	$N(t)$
1.469	0
3.598	71.254
4	62.3383

$\therefore$ The greatest number of vehicles in line at the toll plaza will be 71 vehicles from $1.469 \leq t \leq 4$.

AP Calculus 2022 AB#2 Key

A) $B = 0.781975144055$

$y_B = 1.33024639805$

$$A = \int_{-2}^{B} [f(x) - g(x)]\,dx$$

$$\underline{\underline{A = 3.60354}}$$

B) $h(x) = f(x) - g(x)$

$h'(-0.5) = -0.6$

$\therefore$ h is decreasing at $x = -0.5$ since $h'(x) < 0$ here.

C) $$V = \int_{-2}^{B} [f(x) - g(x)]^2\,dx = \underline{\underline{5.34010}}$$

D) Cross-section $A = [f(x) - g(x)]^2$

let $x = k$ be vertical line and $\frac{dk}{dt} = 7$

$$A'(k) = 2[f(k) - g(k)](f'(k) - g'(k))\frac{dk}{dt}$$

$$A'(-.5) = 2[f(-.5) - g(-.5)][f'(-.5) - g'(-.5)](7)$$

$$= 2(2.20758146)(0.4 - 1)(7)$$

$$= -\underline{\underline{9.27184}}$$

AP Calculus 2022 AB #3 Key

A) $f(0) = \int_0^4 f'(t)\,dt = f(4) - f(0)$

$-2\pi = 3 - f(0)$

$\underline{\underline{f(0) = 3 + 2\pi}}$

$f(5) = \int_4^5 f'(t)\,dt = f(5) - f(4)$

$\frac{1}{2} = f(5) - 3$

$\underline{\underline{f(5) = 3.5}}$

B) POI at $x=2$ because f' changes from decreasing to increasing.

POI at $x=6$ because f' changes from increasing to decreasing.

C) $g'(x) = f'(x) - 1$

$g'(x) = 0$ at $x=5$ and $x=7$

x	0	2	4	5	6	7
$g'(x)$	-1	-3	-1	0	1	0

g is decreasing on the interval $0 \le x \le 5$ since $g'(x) < 0$.

D) By candidates test

x	0	5	7
$g(x)$	$3+2\pi$	-1.5	-0.5

Abs. minimum value is -1.5 because $g'(5) = 0$ and g' changes from negative to positive.

x	0	4	5	6	7
$f(x)$	$3+2\pi$	3	3.5	5	6.5

AP Calculus 2022 AB #4 Key

A) $r''(8.5) = \frac{r'(10)-r'(7)}{10-7} = \frac{-3.8+4.4}{3} = \underline{0.2 \frac{\text{cm}}{\text{day}^2}}$

B) r is twice differentiable, so r' is differentiable therefore continuous. By the Intermediate Value Theorem there exists a value of t on the interval $0 \le t \le 3$ where $r'(t) = -6$ since $r'(0) = -6.1$ and $r'(3) = -5$ and $-6.1 < -6 < -5$.

c) $\int_0^{12} r'(t)\,dt = 3(-5)+4(-4.4)+3(-3.8)+2(-3.5)$

$= -15 - 17.6 - 11.4 - 7$

$= \underline{-51 \text{ cm}}$

D) at $t=3$, $r=100$, $h=50$, $\frac{dh}{dt} = -2$ $\frac{dr}{dt} = -5$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$$

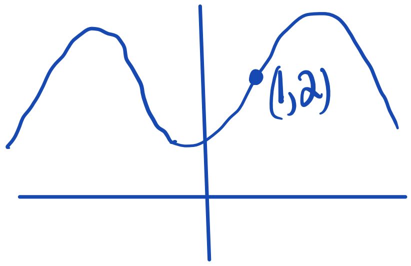
$$= \frac{\pi}{3}\left[2(100)(50)(-5) + 100^2(-2)\right]$$

$$= \frac{\pi}{3}\left[-50{,}000 - 20{,}000\right]$$

$$= \underline{-\frac{70{,}000}{3}\pi \ \frac{\text{cm}^3}{\text{day}}}$$

AP Calculus 2022 AB#5 Key

A)



B) $\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{3}{2}$

Tangent Line
$y = 2 + \frac{3}{2}(x-1)$

$f(0.8) = 2 + \frac{3}{2}(0.8-1)$
$= 2 + \frac{3}{2}\left(\frac{-2}{10}\right)$
$= 2 - 3/10$
$\underline{\underline{f(0.8) = 1.7}}$

C) $f''(1) > 0$, f is concave up on $0.8 < x < 1$ so tangent line lies below curve and $f(0.8)$ underestimates.

D) $\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) dx$

$2\sqrt{y+7} = -\frac{1}{\pi}\cos\left(\frac{\pi}{2}x\right) + C$

$\sqrt{y+7} = -\frac{1}{2\pi}\cos\left(\frac{\pi}{2}x\right) + \frac{6}{2}$

$\underline{\underline{y = \left(\frac{-1}{2\pi}\cos\frac{\pi}{2}x + 3\right)^2 - 7}}$

at (1,2)
$2\sqrt{9} = -\frac{1}{\pi}\cos\frac{\pi}{2} + C$
$\underline{\underline{C = 6}}$

AP Calculus 2022 AB#6 Key

A) $V_P(t) = x_P'(t) = \underline{\underline{4e^{-t}}}$

B) $a_Q(t) = V_Q'(t) = \underline{\underline{\dfrac{-2}{t^3}}}$

$V_Q(t) > 0$ for all $t > 0$ and $a_Q(t) < 0$ for $t > 1$

$\therefore$ Speed of Q is decreasing for all $t > 0$ since $V_Q > 0$ and $a_Q < 0$ (opposite signs).

C) $y_Q(t) = y_Q(1) + \int_1^t \left(\frac{1}{x^2}\right) dx$

$= 2 - \frac{1}{x}\Big|_1^t$

$= 2 - \left(\frac{1}{t} - \frac{1}{1}\right)$

$\underline{\underline{y_Q(t) = 3 - \frac{1}{t}}}$

D) $\lim\limits_{t\to\infty} x_P(t) = 6$ and $\lim\limits_{t\to\infty} y_Q(t) = 3$

$\therefore$ Particle P will be farther from the origin as $t \to \infty$.