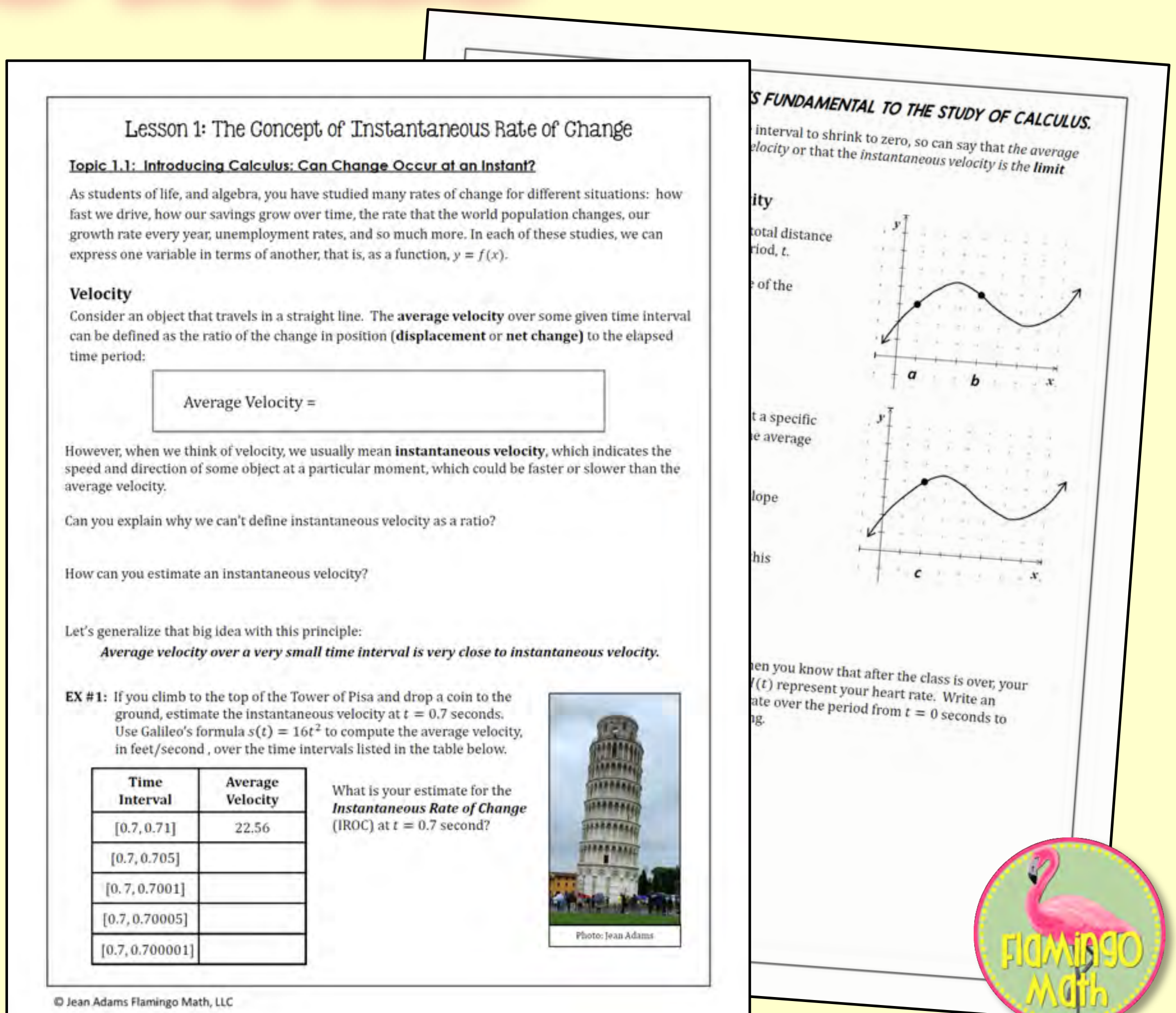
INSTANTANEOUS RATES OF CHANGE

GUIDED NOTES



Lesson 1: The Concept of Instantaneous Rate of Change

Topic 1.1: Introducing Calculus: Can Change Occur at an Instant?

As students of life, and algebra, you have studied many rates of change for different situations: how fast we drive, how our savings grow over time, the rate that the world population changes, our growth rate every year, unemployment rates, and so much more. In each of these studies, we can express one variable in terms of another, that is, as a function, $y = f(x)$.

Velocity

Consider an object that travels in a straight line. The **average velocity** over some given time interval can be defined as the ratio of the change in position (**displacement** or **net change)** to the elapsed time period:

Average Velocity =

However, when we think of velocity, we usually mean **instantaneous velocity**, which indicates the speed and direction of some object at a particular moment, which could be faster or slower than the average velocity.

Can you explain why we can't define instantaneous velocity as a ratio?

How can you estimate an instantaneous velocity?

Let's generalize that big idea with this principle:

Average velocity over a very small time interval is very close to instantaneous velocity.

EX #1: If you climb to the top of the Tower of Pisa and drop a coin to the ground, estimate the instantaneous velocity at $t = 0.7$ seconds. Use Galileo's formula $s(t) = 16t^2$ to compute the average velocity, in feet/second, over the time intervals listed in the table below.

Time Interval	Average Velocity
$[0.7, 0.71]$	22.56
$[0.7, 0.705]$	
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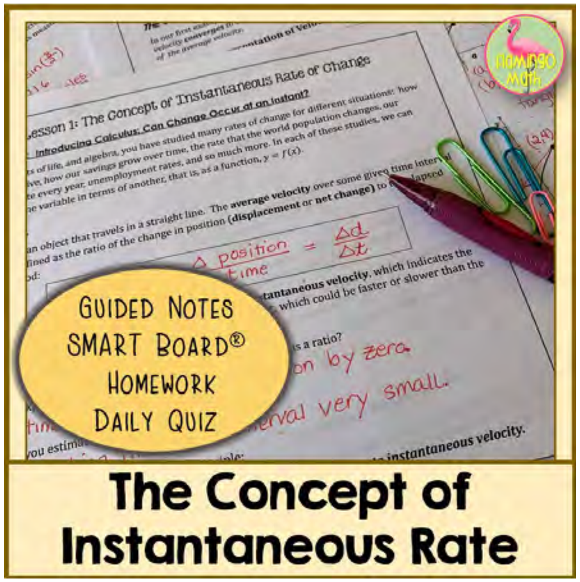
What is your estimate for the ***Instantaneous Rate of Change*** (IROC) at $t = 0.7$ second?

Photo: Jean Adams

...S FUNDAMENTAL TO THE STUDY OF CALCULUS.



The Concept of Instantaneous Rate of Change

Your AP Calculus students will interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.

Try my FREE set of **Guided Student Notes** for the new AP College Board topics. Each of my lessons will provide your students with rigorous examples related to the newest AP Calculus AB and BC curriculum concepts. Teachers also have the benefit of a ***fully-editable*** **SMART Board® Lesson** for presentation and discussion, along with homework assignments, content quizzes, activities, and editable assessments for time-saving lesson preparation.

College Board® Topics:

Topic 1.1 Introducing Calculus: Can Change Occur at an Instant?

College Board® Learning Objectives:

CHA-1 Calculus allows us to generalize knowledge about motion to diverse problems involving change.
CHA-1.A Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.

Teaching Suggestions:

- This is a free set of guided notes for **Unit 1, Lesson 1 Concept of Instantaneous Rate of Change** in my AP Calculus AB Curriculum products. Having taught for 23 years in Central Florida, I found that giving my students a set of scaffolded notes for them to complete during the lesson provided them with a guide for success throughout the year.
- I hope you find the lesson comprehensive and complete.

Thanks for downloading my product!
Be sure to follow me and leave feedback!

Jean

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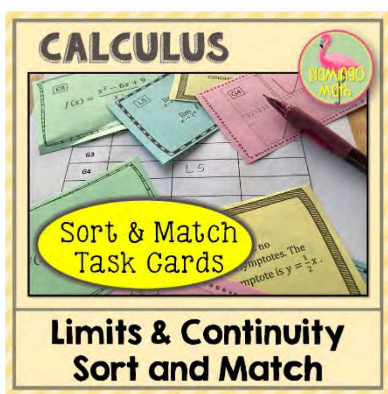

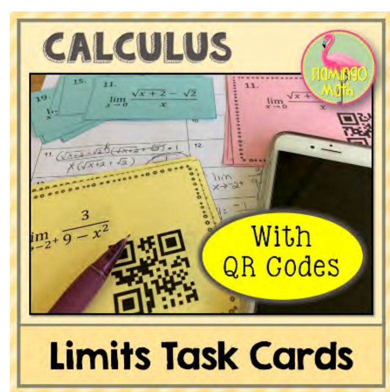

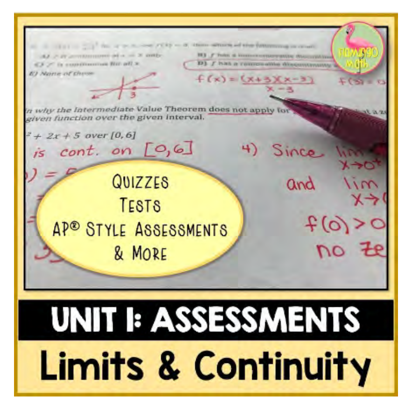

Lesson 1: The Concept of Instantaneous Rate of Change

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Can you explain why we can't define instantaneous velocity as a ratio?

How can you estimate an instantaneous velocity?

Let's generalize that big idea with this principle:

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EX #1: If you climb to the top of the Tower of Pisa and drop a coin to the ground, estimate the instantaneous velocity at $t = 0.7$ seconds. Use Galileo's formula $s(t) = 16t^2$ to compute the average velocity, in feet/second , over the time intervals listed in the table below.

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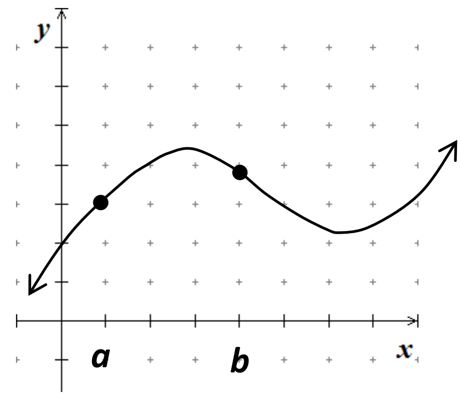
What is your estimate for the ***Instantaneous Rate of Change*** (IROC) at $t = 0.7$ second?

Photo: Jean Adams

THE STUDY OF RATE OF CHANGE IS FUNDAMENTAL TO THE STUDY OF CALCULUS.

In our first example, we allowed the time interval to shrink to zero, so can say that *the average velocity* ***converges*** *to the instantaneous velocity* or that the *instantaneous velocity is the* ***limit*** *of the average velocity.*

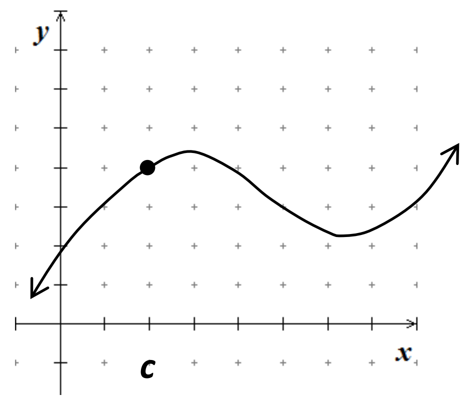
Graphical Representation of Velocity



Average Speed (AROC) is defined by the total distance traveled, d, divided by the elapsed time period, t.

Graphically, the average speed is the slope of the secant line.

Analytically, average speed AROC $= \frac{\Delta d}{\Delta t}$



Instantaneous Speed (IROC) is the speed at a specific instant in time, where $t = t_0$ is the limit of the average rates of change.

Graphically, the instantaneous speed is the slope of the tangent line at a specific point.

Analytically, we need ***calculus*** to determine this instantaneous speed.

EX #2: Spin Class Heart Rate and Exercise

Have you ever taken a Spin Class at the gym? Then you know that after the class is over, your rapid heart rate decreases as time passes. Let $H(t)$ represent your heart rate. Write an expression for the rate of change of your heart rate over the period from $t = 0$ seconds to $t = 90$ seconds, after you have stopped exercising.

EX #3: Roller Coaster

A **hypercoaster** is a special circuit roller coaster with a height measuring greater than 200 feet. If a new hypercoaster is under design to have its largest drop on the circuit modeled by the equation below, where $d(t)$ is measured in feet, and time t, is measured in seconds.

$$d(t) = 1.87t^3 - 20.64t^2 + 112.79t + 132.56$$

A. Find the average speed of the hypercoaster from 2 seconds to 8 seconds.

B. What is the average speed of the hypercoaster from 4 seconds to 6 seconds?

C. Estimate the instantaneous speed of the hypercoaster at exactly 5 seconds.

Time Interval	Average Velocity
$[5, 5.1]$	
$[5, 5.01]$	
$[5, 5.001]$	

D. Explain the meaning of the instantaneous speed in the context of the question at $t = 5$ seconds.

EX #4: Airport Departures

The traffic pattern of departing flights at Orlando International Airport on a fall afternoon can be modeled by the function V defined by $V(t) = 92 - 15\sin\left(\frac{t}{3}\right)$ where $V(t)$ is measured in vehicles and t is measured in minutes $0 \leq t \leq 30$.

A. Find $V(t)$ for $t = 18$.

B. Make a sketch of the graph of time *vs*. number of vehicles. Be sure your graph has a scale.

C. What is the average rate of change of the vehicles at the departing flights over the time interval $15 \leq t \leq 20$? Indicate units of measure.

D. Estimate the instantaneous rate of change of vehicles at $t = 18$ by finding the average rates from $t = 18$ to $t = 18.1$, $t = 18$ to $t = 18.01$, and $t = 18$ to $t = 18.001$.

Time Interval	**Average Velocity**
$[18, 18.1]$	
$[18, 18.01]$	
$[18, 18.001]$	

E. Why can't the instantaneous rate of change of traffic in the departure lane with respect to time be calculated using the method in part D?

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Velocity

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$$\text{Average Velocity} = \frac{\Delta \text{ position}}{\Delta \text{ time}} = \frac{\Delta d}{\Delta t}$$

However, when we think of velocity, we usually mean **instantaneous velocity**, which indicates the speed and direction of some object at a particular moment, which could be faster or slower than the average velocity.

Can you explain why we can't define instantaneous velocity as a ratio?

The time interval is zero, division by zero.

How can you estimate an instantaneous velocity?

By making the time interval very small.

Let's generalize that big idea with this principle:

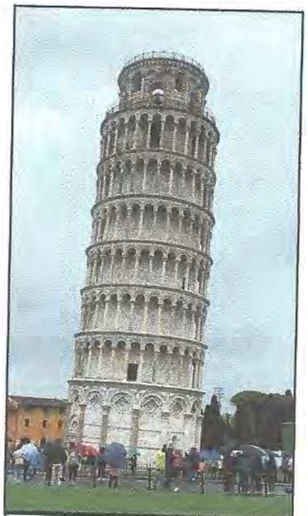
Average velocity over a very small time interval is very close to instantaneous velocity.

EX #1: If you climb to the top of the Tower of Pisa and drop a coin to the ground, estimate the instantaneous velocity at $t = 0.7$ seconds. Use Galileo's formula $s(t) = 16t^2$ to compute the average velocity, in feet/second, over the time intervals listed in the table below.

Time Interval	Average Velocity
[0.7, 0.71]	22.56
[0.7, 0.705]	22.48
[0.7, 0.7001]	22.4016
[0.7, 0.70005]	22.4008
[0.7, 0.700001]	22.40016

What is your estimate for the ***Instantaneous Rate of Change*** (IROC) at $t = 0.5$ second?

The coin is falling at a rate of about 22.400 ft/sec at $t = 0.7$ second.



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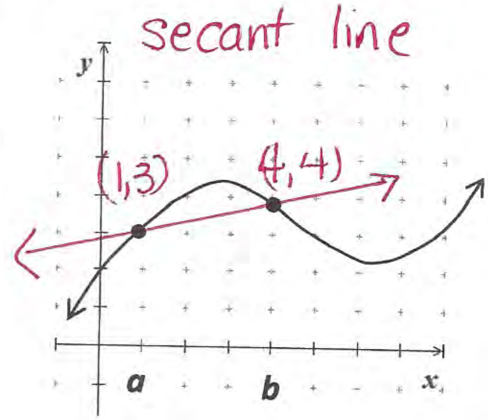
Graphical Representation of Velocity

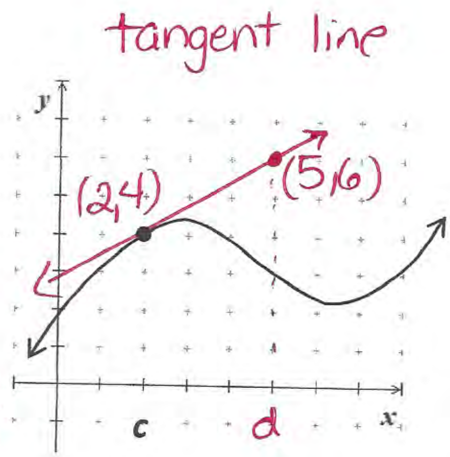


Average Speed (AROC) is defined by the total distance traveled, d, divided by the elapsed time period, t.

Graphically, the average speed is the slope of the secant line.

Analytically, average speed AROC $= \frac{\Delta d}{\Delta t}$

$$AROC = m_{sec} = \frac{4-3}{4-1} = \frac{1}{3}$$



Instantaneous Speed (IROC) is the speed at a specific instant in time, where $t = t_0$ is the limit of the average rates of change.

Graphically, the instantaneous speed is the slope of the tangent line at a specific point.

Analytically, we need ***calculus*** to determine this instantaneous speed.

Estimate IROC at (2,4) is about $\frac{2}{3}$

EX #2: Spin Class Heart Rate and Exercise

Have you ever taken a Spin Class at the gym? Then you know that after the class is over, your rapid heart rate decreases as time passes. Let $H(t)$ represent your heart rate. Write an expression for the rate of change of your heart rate over the period from $t = 0$ seconds to $t = 90$ seconds, after you have stopped exercising.

$$\frac{\Delta H}{\Delta t} = \frac{H(90) - H(0)}{90 - 0} = \frac{H(90) - H(0)}{90}$$

EX #3: Roller Coaster

A **hypercoaster** is a special circuit roller coaster with a height measuring greater than 200 feet. If a new hypercoaster is under design to have its largest drop on the circuit modeled by the equation below, where $d(t)$ is measured in feet, and time t, is measured in seconds.

$$d(t) = 1.87t^3 - 20.64t^2 + 112.79t + 132.56$$

A. Find the average speed of the hypercoaster from 2 seconds to 8 seconds.

$$\text{AROC} = \frac{d(8) - d(2)}{8-2} \approx \frac{671.36 - 290.54}{6}$$

AROC on $2 \le t \le 8$ sec. is about 63.47 ft/sec.

B. What is the average speed of the hypercoaster from 4 seconds to 6 seconds?

$$\text{AROC} = \frac{d(6) - d(4)}{6-4} \approx \frac{470.18 - 373.16}{2}$$

AROC on $4 \le t \le 6$ sec. is about 48.51 ft/sec

C. Estimate the instantaneous speed of the hypercoaster at exactly 5 seconds.

Time Interval	Average Velocity
$[5, 5.1]$	47.3997 ft/sec
$[5, 5.01]$	46.7142 ft/sec
$[5, 5.001]$	46.6474 ft/sec

The instantaneous speed of the hyper-coaster at 5 seconds is about 46.647 ft/sec.

D. Explain the meaning of the instantaneous speed in the context of the question at $t = 5$ seconds.

The rate of change in the drop of the hypercoaster at $t=5$ seconds is about 46.647 ft per second.

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The traffic pattern of departing flights at Orlando International Airport on a fall afternoon can be modeled by the function V defined by $V(t) = 92 - 15\sin\left(\frac{t}{3}\right)$ where $V(t)$ is measured in vehicles and t is measured in minutes $0 \le t \le 30$.

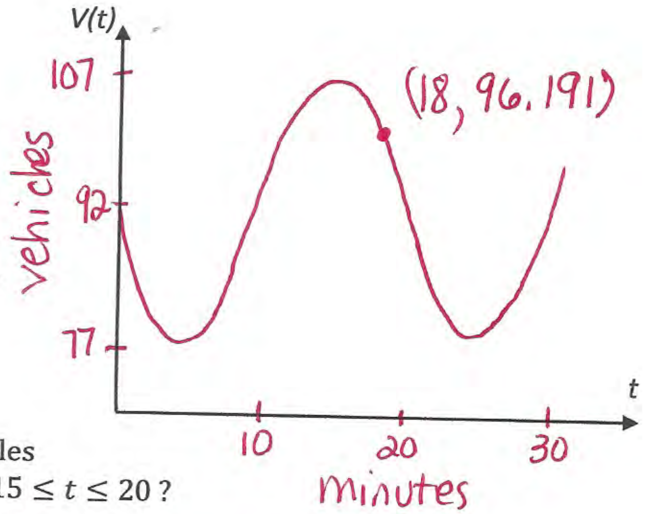
A. Find $V(t)$ for $t = 18$.

$V(18) = 92 - 15\sin\left(\frac{18}{3}\right)$

$V(18) \approx 96.1912$

about 96 vehicles

B. Make a sketch of the graph of time *vs.* number of vehicles. Be sure your graph has a scale.



C. What is the average rate of change of the vehicles at the departing flights over the time interval $15 \le t \le 20$? Indicate units of measure.

$$AROC = \frac{V(20) - V(15)}{20 - 15} \approx \frac{86.3877 - 106.3838}{5} \approx -3.9992 \frac{veh}{min}$$

Average rate of change on [15, 20] minutes is decreasing at a rate of about 4 vehicles per minute.

D. Estimate the instantaneous rate of change of vehicles at $t = 18$ by finding the average rates from $t = 18$ to $t = 18.1$, $t = 18$ to $t = 18.01$, and $t = 18$ to $t = 18.001$.

Time Interval	Average Velocity
[18, 18.1]	$-4.823 \frac{veh}{min}$
[18, 18.01]	$-4.803 \frac{veh}{min}$
[18, 18.001]	$-4.801 \frac{veh}{min}$

The instantaneous rate of change at $t = 18$ minutes is decreasing at a rate of about 4.8 vehicles per minute

E. Why can't the instantaneous rate of change of traffic in the departure lane with respect to time be calculated using the method in part D?

division by zero would occur.