AP CALCULUS AB

STUFF YOU MUST KNOW COLD

HANDY REFERENCE SHEET FOR THE AP CALC EXAM

Flamingo Math

THANKS FOR DOWNLOADING MY PRODUCTS!

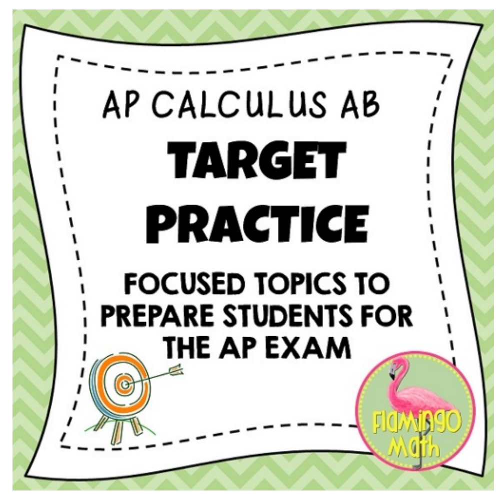
As you prepare your students for the AP Calculus AB Exam, here's an adaptation to Sean Bird's "Stuff You Must Know Cold" handout. (http://covenantchristian.org/bird/Calculus.htm) Formulas, Theorems, and Other concepts necessary for success for the Calculus AB student. You will find additional resources, including a PowerPoint version, a Word document in color, as well as AP Calculus BC topics on his website.

If you like this product, you might enjoy my Calculus Target Practice. Follow this link to preview the packet

http://www.teacherspayteachers.com/Product/AP-Calculus-AB-Target-Practice-1210286

You might want to visit my store for more engaging lessons and review materials.

http://www.teacherspayteachers.com/Store/Jean-Adams



Credits:

Polka Dot Background

STUFF YOU MUST KNOW COLD . . .

Alternate Definition of the Derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos\ x$$

$$\frac{d}{dx}(\cos x) = -\sin\ x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

Where u is a function of x,
and a is a constant.

Differentiation Rules

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}\ \ OR\ \ \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule:

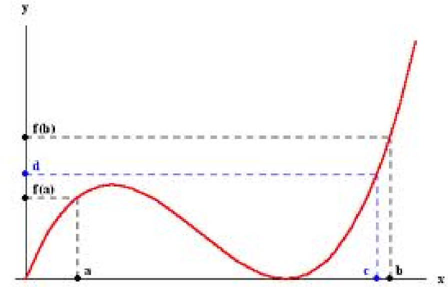
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}\ \ OR\ u\,v' + v\,u'$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\ \ OR\ \ \frac{v\,u' - u\,v'}{v^2}$$

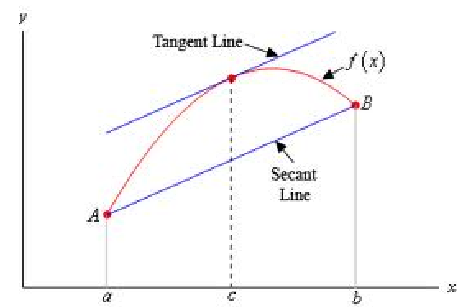
Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x = c$ in the open interval (a, b) such that $f(c) = y$.



Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *__the first derivative exists on the interval__* (a, b) then there is at least one number $x = c$ in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

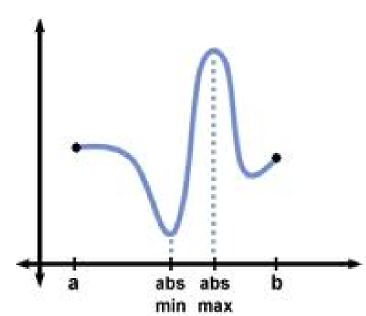


Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, **AND** *__the first derivative exists on the interval__* (a, b) **AND** $f(a) = f(b)$, then there is at least one number $x = c$ in (a, b) such that $f'(c) = 0$.

Extreme Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, then the function is guaranteed to have an absolute maximum and an absolute minimum on the interval.



Derivative of an Inverse Function:

If f has an inverse function g then:

$$g'(x) = \frac{1}{f'(g(x))}$$

derivatives are reciprocal slopes

Implicit Differentiation

Remember that in implicit differentiation you will have a $\frac{dy}{dx}$ for each y in the original function or equation. Isolate the $\frac{dy}{dx}$. If you are taking the second derivative $\frac{d^2y}{dx^2}$, you will often substitute the expression you found for the first derivative somewhere in the process.

Average Rate of Change ARoC:

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change IRoC:

$$m_{tan} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Curve Sketching And Analysis

$y = f(x)$ must be continuous at each:

Critical point: $\frac{dy}{dx} = 0$ or undefined

LOOK OUT FOR ENDPOINTS

Local minimum:

$\frac{dy}{dx}$ goes $(-, 0, +)$ or $(-, und, +)$ OR $\frac{d^2y}{dx^2} > 0$

Local maximum:

$\frac{dy}{dx}$ goes $(+, 0, -)$ or $(+, und, -)$ OR $\frac{d^2y}{dx^2} < 0$

Point of inflection: concavity changes

$\frac{d^2y}{dx^2}$ goes from $(+, 0, -)$, $(-, 0, +)$, $(+, und, -)$, OR $(-, und, +)$

First Derivative:

$f'(x) > 0$ function is increasing.

$f'(x) < 0$ function is decreasing.

$f'(x) = 0$ or DNE: Critical Values at x.

Relative Maximum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $+$ to $-$.

Relative Minimum: $f'(x) = 0$ or DNE and sign of $f'(x)$ changes from $-$ to $+$.

Absolute Max or Min:

MUST CHECK ENDPOINTS ALSO

The maximum value is a y-value.

Second Derivative:

$f''(x) > 0$ function is concave up.

$f''(x) < 0$ function is concave down.

$f'(x) = 0$ and sign of $f''(x)$ changes, then there is a point of inflection at x.

Relative Maximum: $f''(x) < 0$

Relative Minimum: $f''(x) > 0$

Write the equation of a tangent line at a point:

You need a slope (derivative) and a point.

$$y_2 - y_1 = m(x_2 - x_1)$$

Horizontal Asymptotes:

1. If the largest exponent in the numerator is < largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = 0$.

2. If the largest exponent in the numerator is > the largest exponent in the denominator then $\lim_{x \to \pm\infty} f(x) = DNE$

3. If the largest exponent in the numerator is = to the largest exponent in the denominator then the quotient of the leading coefficients is the asymptote. $\lim_{x \to \pm\infty} f(x) = \frac{a}{b}$

ONLY FOUR THINGS YOU CAN DO ON A CALCULATOR THAT NEEDS NO WORK SHOWN:

1. Graphing a function within an arbitrary view window.
2. Finding the zeros of a function.
3. Computing the derivative of a function numerically.
4. Computing the definite integral of a function numerically.

Distance, Velocity, and Acceleration

$x(t)$ = position function
$v(t)$ = velocity function
$a(t)$ = acceleration function

The derivative of position (ft) is velocity (ft/sec); the derivative of velocity (ft/sec) is acceleration (ft/sec^2).

The integral of acceleration (ft/sec^2) is velocity (ft/sec) ; the integral of velocity (ft/sec) is position (ft).

Speed is | velocity |

If acceleration and velocity have the ***same sign***, then the speed is *increasing*..

If the acceleration and velocity have ***different signs***, then the speed is *decreasing.*

The particle is moving right when velocity is positive and particle is moving left when velocity is negative.

Displacement = $\int_{t_0}^{t_f} v(t)\,dt$

Total Distance $= \int_{initial\ time}^{final\ time} |\,v(t)|\ dt$

Average Velocity

$$= \frac{\text{final position} - \text{initial position}}{\text{total time}} = \frac{\Delta x}{\Delta t}$$

Accumulation = $x(0) + \int_{t_0}^{t_f} v(t)\,dt$

LOGARITHMS
Definition:

$$ln\ N = p \leftrightarrow e^p = N$$

$ln\ e = 1$
$\ln 1 = 0$
$\ln(MN) = \ln M + \ln N$
$\ln\left(\frac{M}{N}\right) = \ln M - \ln N$
$p \cdot \ln M = \ln M^p$

EXPONENTIAL GROWTH and DECAY:

When you see these words use: $y = Ce^{kt}$

"y is a differentiable function of t such that $y > 0$ *and* $y' = ky$ *"*

"the rate of change of y is proportional to y"

When solving a differential equation:

1. Separate variables first
2. Integrate
3. Add +C to one side
4. Use initial conditions to find "C"
5. Write the equation if the form of $y = f(x)$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where $F'(x) = f(x)$

Corollary to FTC

$$\frac{d}{dx}\int_a^{g(u)} f(t)dt = f\big(g(u)\big)\frac{du}{dx}$$

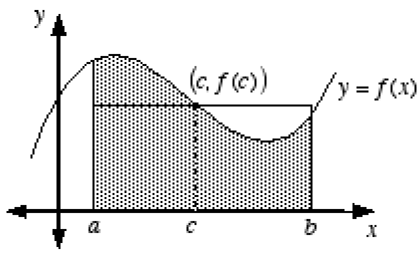
Mean Value Theorem for Integrals: The Average Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{\int_a^b f(x)dx}{b-a}$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

The rectangle has the same area as the shaded region under the curve.



Riemann Sums

A Riemann Sum means a rectangular approximation. Approximation means that you ***DO NOT EVALUATE THE INTEGRAL***; you add up the areas of the rectangles.

Trapezoidal Rule

For uneven intervals, may need to calculate area of one trapezoid at a time and total.

$A_{Trap} = \frac{1}{2}h[b_1 + b_2]$

For even intervals:

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[\begin{matrix} y_0 + 2y_1 + 2y_2 + \ldots \\ +2y_{n-1} + y_n \end{matrix}\right]$$

Values of Trigonometric Functions for Common Angles

θ	$sin\ \theta$	$cos\ \theta$	$tan\ \theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	"∞"
π	0	-1	0

Must know both inverse trig and trig values:

EX. $tan\ \frac{\pi}{4} = 1$ and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

ODD and EVEN:

$\sin(-x) = -\sin x$ (odd)

$\cos(-x) = \cos x$ (even)

Trigonometric Identities

Pythagorean Identities:

$sin^2\theta + cos^2\theta = 1$

The other two are easy to derive by dividing by $\sin^2\theta$ or $\cos^2\theta$.

$1 + \tan^2\theta = sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Double Angle Formulas:

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

Power-Reducing Formulas:

$\cos^2 x = \frac{1}{2}(1 + \cos 2x\ \ )$

$\sin^2 x = \frac{1}{2}\ (1 - \cos 2x)$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities:

$csc\ x = \frac{1}{sin\ x}$ or $\sin x\csc x = 1$

$sec\ x = \frac{1}{\cos x}$ or $\cos x\sec x = 1$

Basic Integrals

$$\int du = u + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \; n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u \, du = e^u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u + C|$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \sec^2 u \, due = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

Area and Solids of Revolution:

NOTE: $(\boldsymbol{a}, \boldsymbol{b})$ are $\boldsymbol{x}$-coordinates and $(\boldsymbol{c}, \boldsymbol{d})$ are $\boldsymbol{y}$-coordinates

Area Between Two Curves:

Slices ⊥ to x-axis: $A = \int_a^b [f(x) - g(x)] \, dx$

Slices ⊥ to y-axis: $A = \int_c^d [f(y) - g(y)] \, dy$

Volume By Disk Method:

About x-axis: $V = \pi \int_a^b [R(x)]^2 \, dx$

About y-axis: $V = \pi \int_c^d [R(y)]^2 \, dy$

Volume By Washer Method:

About x-axis: $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \, dx$

About y-axis: $V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2) \, dy$

Volume By Shell Method:

About x-axis: $V = 2\pi \int_c^d y \, [R(y)] \, dy$

About y-axis: $V = 2\pi \int_a^b x \, [R(x)] \, dx$

General Equations for Known Cross Section where *base* is the distance between the two curves and *a* and *b* are the limits of integration.

SQUARES: $V = \int_a^b (base)^2 \, dx$

TRIANGLES

EQUILATERAL: $V = \frac{\sqrt{3}}{4} \int_a^b (base)^2 \, dx$

ISOSCELES RIGHT: $V = \frac{1}{4} \int_a^b (base)^2 \, dx$

RECTANGLES: $V = \int_a^b (base) \cdot h \, dx$
where h is the height of the rectangles.

SEMI-CIRCLES: $V = \frac{\pi}{2} \int_a^b (radius)^2 \, dx$
where radius is ½ distance between the two curves.

MORE DERIVATIVES:

$$\frac{d}{dx}\left[\sin^{-1}\frac{u}{a}\right] = \frac{1}{\sqrt{a^2-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}\frac{u}{a}\right] = \frac{a}{a^2+u^2}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\cot^{-1}x\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}\frac{u}{a}\right] = \frac{a}{|u|\sqrt{u^2-a^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left[\csc^{-1}x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(a^u\right) = a^u \ln a\,\frac{du}{dx}$$

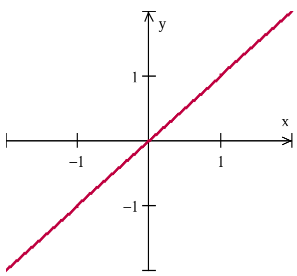
$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x\ln a}$$

MORE INTEGRALS:

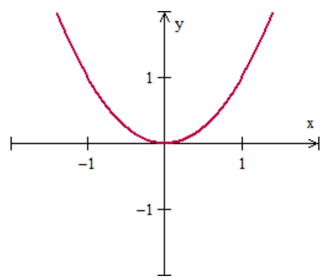
$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

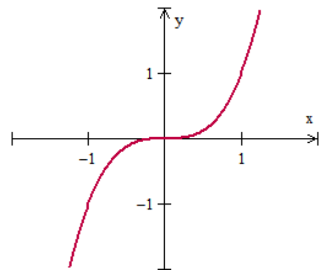
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$$



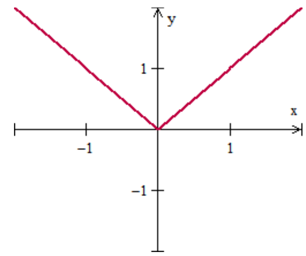
$y = x$



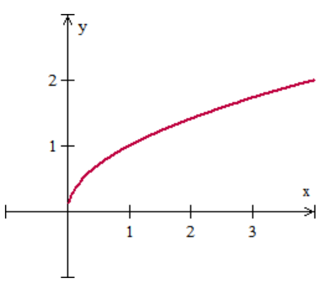
$y = x^2$



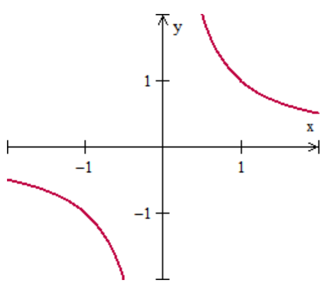
$y = x^3$



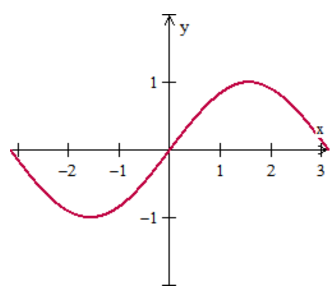
$y = |x|$



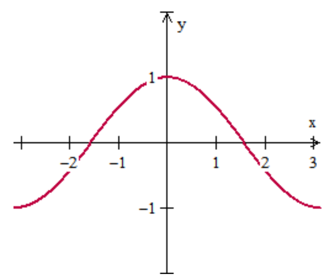
$y = \sqrt{x}$



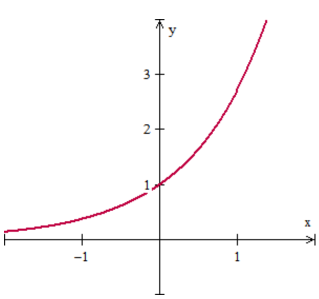
$y = \dfrac{1}{x}$



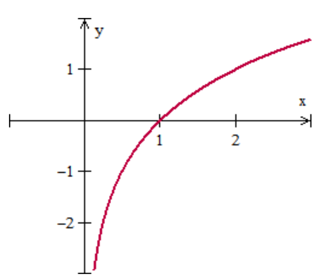
$y = \sin x$



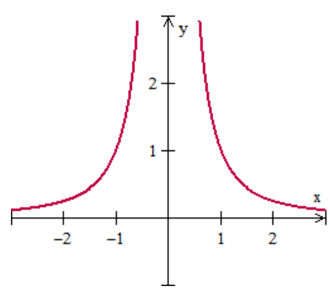
$y = \cos x$



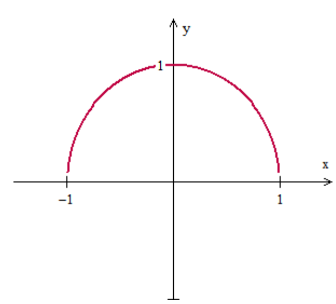
$y = e^x$



$y = \ln x$



$y = \dfrac{1}{x^2}$



$y = \sqrt{a^2 - x^2}$

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