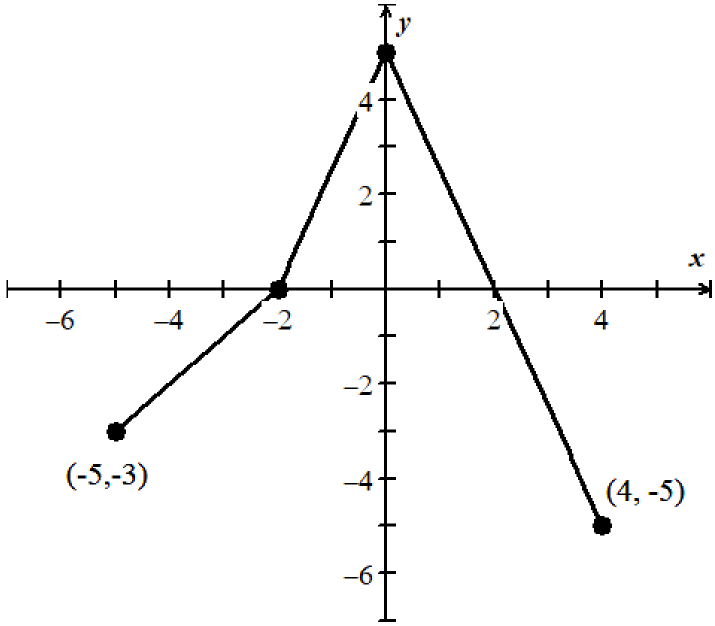
Graph Analysis & Justification Practice

Name ______________________________



Graph of f

The function *f* is defined on the closed interval [−5, 4]. The graph of *f* consists of three line segments and is shown in the figure above. Let *g* be the function defined by

$$g(x) = \int_{-2}^{x} f(t)dt$$

A) Find the $g(2)$, $g'(2)$ and $g''(2)$.

B) For what values of *x* in the open interval $-5 < x < 4$ is *g* decreasing? Explain your reasoning.

C) For what values of *x* in the open interval $-5 < x < 4$ is *g* concave up? Explain your reasoning.

D) Find all values of *x* on the open interval $-5 < x < 4$ for which the function *g* has a local minimum. Justify your answer.

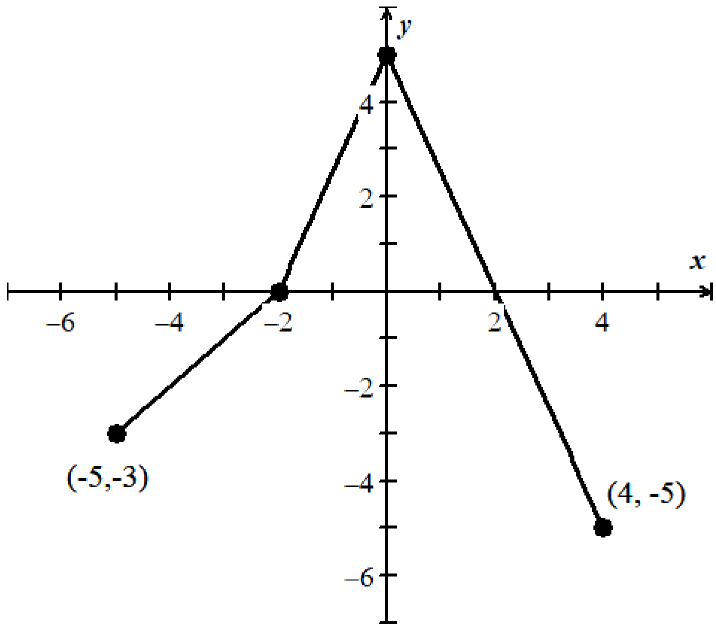
E) On what open intervals contained in $-5 < x < 4$ is the graph of *g* both increasing and concave down? Give a reason for your answer.

F) Does the graph of *g* have a point of inflection at $x = 0$? Justify your answer.

G) Find the absolute minimum value and the absolute maximum value of *g* on the interval $[-5, 4]$. Justify your answers.

H) For $[-5, 4]$, find all intervals for which $g(x) \leq 0$.

Graph Analysis & Justification Practice Key



Graph of f

The function *f* is defined on the closed interval [−5, 4]. The graph of *f* consists of three line segments and is shown in the figure above. Let *g* be the function defined by

$$g(x) = \int_{-2}^{x} f(t)dt$$

KEY:

A) $g(2) = \int_{-2}^{2} f(t)dt = \frac{1}{2}(4)(5) = 10$ $g'(2) = 0$ and $g''(2) = -\frac{5}{2}$

B) *G* decreases where $g'(x) = f(x)$ is negative. G decreases $-5 < x < -2$ and $2 < x < 4$.

C) *G* is concave down where $g'(x) = f(x)$ is decreasing. *f* is decreasing on $0 < x < 4$, so *G* is concave down here.

D) G has a local minimum when *f* changes from negative to positive, so at $x = -2$, *G* has a local minimum.

E) The graph of *G* will be increasing and concave down where $g'(x) = f(x)$ is positive and decreasing. This happens on $0 < x < 2$.

F) *G* has a point of inflection where $f'(x)$ is zero or dne. Candidates are $x = -2$ *and* $x = 0$. Also, where $g'(x) = f(x)$ changes from increasing to decreasing. So $g(x)$ has a point of inflection at $x = 0$ because *f* changes from increasing to decreasing here.

G) The graph of G has an absolute max or absolute min where the derivative changes signs and where g' = f is zero or dne. By the candidates test, $G(-5) = 4.5, g(-2) = 0,$ $g(2) = 10, g(4) = 5$. Absolute max value is 10 absolute min value is 0.

H) $G(x) \geq 0$ on the entire interval and is never negative.

Graph Analysis &
More Justification Practice

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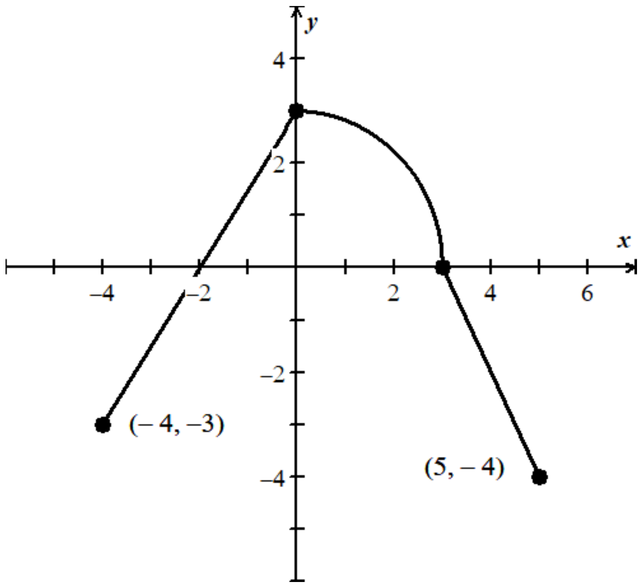


Graph of f'

Let f be a function defined on the closed interval [−4, 5] with $f(0) = -2$. The graph of f', the derivative of f, consists of two line segments and a quarter circle, as shown above.

A) On what intervals, if any, is f increasing? Justify your answer.

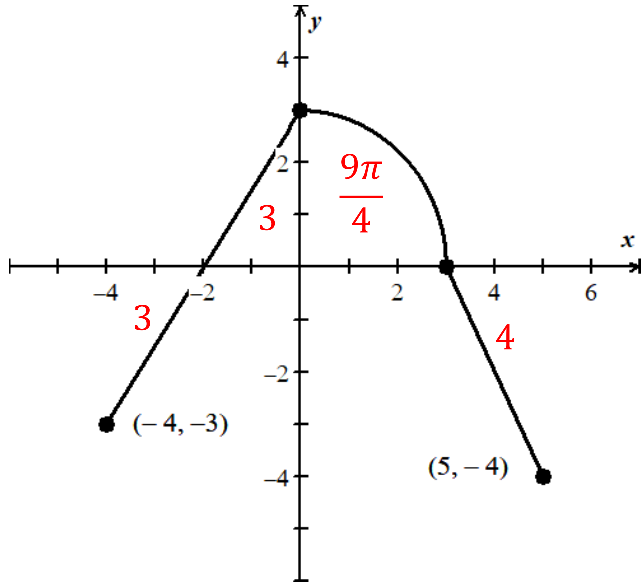
B) For $-4 \leq x \leq 5$, find the value of x at which f has an absolute maximum. Justify your answer.

C) Find the *x*-coordinate of each point of inflection of the graph of f on the open interval $-4 < x < 5$. Justify your answer.

D) Find an equation for the line tangent to the graph of f at the point $(0, -2)$.

E) Find $f(-4)$ and $f(5)$. Show work that leads to your answers.

Graph Analysis & More Justification Practice Key



Graph of f'

Let f be a function defined on the closed interval [−4, 5] with $f(0) - 2$. The graph of f', the derivative of f, consists of two line segments and a quarter circle, as shown above.

A) On what intervals, if any, is f increasing? Justify your answer.

f is increasing on $-2 \le x \le 3$ because $f'(x) > 0$ on this interval.

B) For $-4 \le x \le 5$, find the value of x at which f has an absolute maximum. Justify your answer.

On $-4 \le x \le 5$, f would obtain a maximum where $f'(x)$ changes from positive to negative. This occurs at $x = 3$.

C) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-4 < x < 5$. Justify your answer.

Where $f'(x)$ changes from increasing to decreasing there is a point of inflection so, at $x = 0$.

D) Find an equation for the line tangent to the graph of f at the point $(0, -2)$.

$f'(0) = 3$ and $(0, -2)$ Tangent line: $y = 3x - 2$

E) Find $f(-4)$ and $f(5)$. Show work that leads to your answers.

$f(-4)$ is: IC + $\int_{-4}^{0} f'(t)dt$: -2 + area $(-3 + 3) = -2$

$f(5)$ is: IC + $\int_{0}^{5} f'(t)dt$ -2 + area $\left(\frac{9\pi}{4} - 4\right) = -6 + \frac{9\pi}{4}$