FOCUS TOPIC 1: Rate & Accumulation

Name:____________________________________

What students should be able to do:

- Be ready to read and apply; often these problems contain a lot of writing which needs to be carefully read.
- Recognize that rate = derivative.
- Recognize a rate from the units given without the words "rate" or "derivative."
- Find the change in an amount by integrating the rate. The integral of a rate of change gives the amount of change (FTC):

$$\int_a^b f'(t)dt = f(b) - f(a)$$

- Find the final amount by adding the initial amount to the amount found by integrating the rate. If $x = x_0$ is the initial time, and $f(x_0)$ is the initial amount, then final accumulated amount is

$$f(x) = f(x_0) + \int_{x_0}^{x} f'(t)dt$$

- Understand the question. It is often not necessary to do as much computation as it seems at first.
- Use FTC to differentiate a function defined by an integral.
- Explain the meaning of a derivative or its value in terms of the context of the problem.
- Explain the meaning of a definite integral or its value in terms of the context of the problem. The explanation should contain
  (1) what it represents,
  (2) its units, and
  (3) how the limits of integration apply in context.
- Store functions in their calculator recall them to do computations on their calculator.
- If the rates are given in a table, be ready to approximate an integral using a Riemann sum or by trapezoids.
- Do a max/min or increasing/decreasing analysis.

NOTES:

FOCUS TOPIC 2: Linear Motion

Name:________________________________

What students should be able to do:

- Understand and use the relationships above.
- Distinguish between position at some time and the total distance traveled during the time period.
- The total distance traveled is the definite integral of the speed

$$\int_a^b |v(t)|\, dt$$

- The net distance traveled, displacement, is the definite integral of the velocity (rate of change):

$$\int_a^b v(t)\, dt$$

- Note that "displacement" has not been used preciously on AP exam, but (as per the new *Course and Exam Description*) may be used now. Be sure your students know this term.
- The final position is the initial position plus the definite integral of the rate of change from $x= a$ to $x = t$:

$$s(t) = s(a) + \int_a^t v(x)\, dx$$

<u>Notice that this is an accumulation function equation (Type 1).</u>

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position or velocity. These are easily handled with the accumulation equation in the bullet above.
- Find the speed at a given time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration
- Determine if the speed is increasing or decreasing.
- If at some time, the velocity and acceleration have the *same* sign then the speed is increasing. If they have *different* signs the speed is decreasing.
- If the velocity graph is moving away from (towards) the t-axis the speed is increasing (decreasing).
- Use a difference quotient to approximate derivative.
- Riemann sum approximations.

Units of measure.

- Interpret meaning of a derivative or a definite integral in context of the problem

NOTES:

FOCUS TOPIC 3: Graph Analysis

(Here's the graph of the derivative, tell me something about the function…)

Name:____

What students should be able to do:

- Read information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find where the function is increasing or decreasing.
- Find and justify extreme values (1st and 2nd derivative tests, Closed interval test aka. Candidates' test).
- Find and justify points of inflection.
- Find slopes (second derivatives, acceleration) from the graph.
- Write an equation of a tangent line.
- Evaluate Riemann sums from geometry of the graph only.
- FTC: Evaluate integral from the area of regions on the graph.
- FTC: The function, $g(x)$, maybe defined by an integral where the given graph is the graph of the integrand, $f(t)$, so students should know that if,

  $$g(x) = g(a) + \int_a^t f(t)\,dt$$

  then $g'(x) = f(x)$ and $g''(x) = f'(x)$. In this case students should write $g'(t) = f(t)$ on their answer paper, so it is clear to the reader that they understand this.

NOTES:

FOCUS TOPIC 4: Area & Volume

Name:______________________________

What students should be able to do:

- Find the intersection(s) of the graphs and use them as limits of integration (calculator equation solving). Write the equation followed by the solution; showing work is not required. Usually no credit is earned until the solution is used in context (as a limit of integration). Students should know how to store and recall these values to save time and avoid copy errors.
- Find the area of the region between the graph and the *x*-axis or between two graphs.
- Find the volume when the region is revolved around a line, not necessarily an axis or an edge of the region, by the disk/washer method.
- The cylindrical shell method will *never be necessary* for a question on the AP exams, but is eligible for full credit if properly used.
- Find the volume of a solid with regular cross-sections whose base is the region between the curves. For an interesting variation on this idea see 2009 AB 4(b)
- Find the equation of a vertical line that divides the region in half (area or volume). This involves setting up and solving an integral equation where the limit is the variable for which the equation is solved.

NOTES:

FOCUS TOPIC 5: Table & Riemann Sums
Name:___________________________________

What students should be able to do:

- Find the average rate of change over an interval
- Approximate the derivative using a difference quotient. Use the two values closest to the number at which you are approximating. This amounts to finding the slope. Show the quotient even if you can do the arithmetic in your head.
- Use Riemann sums (left, right, midpoint), or a trapezoidal approximation to approximate the value of a definite integral using values in the table (typically with uneven subintervals). The Trapezoidal Rule, *per se*, is not required; it is expected that students will add the areas of a small number of trapezoids without reference to a formula.
- Average value, average rate of change, Rolle's Theorem, the Mean Value Theorem and the Intermediate Value Theorem. (See 2007 AB 3 – four simple parts that could be multiple-choice questions; the mean on this question was 0.96 out of a possible 9.)
- These questions are usually presented in some context and answers should be in that context.
- Unit analysis.

Do's and Don'ts

Do: Remember that you do not know what happens between the values in the table unless some other information is given. For example, don't assume that the largest number in the table is the maximum value of the function.

Do: Show what you are doing even if you can do it in your head. If you're finding a slope, show the quotient.

Do Not do arithmetic: A long expression consisting entire of numbers such as you get when doing a Riemann sum, does not need to be simplified in any way. If you simplify correct answer incorrectly, you will lose credit. However, do not leave expression such as R(3) – pull its numerical value from the table.

Do Not: Find a regression equation and then use that to answer parts of the question. While regression is perfectly good mathematics, regression equations are not one of the four things students may do with their calculator. Regression gives only an approximation of our function. The exam is testing whether students can work with numbers.

NOTES:

FOCUS TOPIC 6: Differential Equations & Slope Fields

Name:_____________________________________

What students should be able to do:

- Find the *general solution* of a differential equation using the method of separation of variables (this is the *only* method tested).
- Find a *particular solution* using the initial condition to evaluate the constant of integration – initial value problem (IVP).
- **NEW** Determine the domain restrictions on the solution of a differential equation.
- Understand that proposed solution of a differential equation is a function (not a number) and if it and its derivative are substituted into the given differential equation the resulting equation is true. This may be part of doing the problem even if solving the differential equation is not required
- Growth-decay problems.
- Draw a slope field by hand.
- Sketch a *particular solution* on a (given) slope field.
- Interpret a slope field.
- Use the given derivative to analyze a function such as finding extreme values

<u>The five points are now distributed this way:</u>

- one point for separating the variables
- one point each for finding the antiderivatives
- one point for including the constant of integration and using the initial condition – that is, for writing "+ C" on the paper with one of the antiderivatives and substituting the initial condition; finding is value is included in the "answer point." and
- one point, the "answer point", for the correct answer. This point includes all the algebra and arithmetic in the problem.

NOTES:

FOCUS TOPIC 7: Others
(Implicit Differentiation, Related Rates, Theorems, etc.)
Name:________________________________

Implicitly defined functions
What students should know how to do:

- Know how to find the first derivative of an implicit relation using the product rule, quotient rule, the chain rule, etc.
- Know how to find the second derivative, including substituting for the first derivative.
- Know how to evaluate the first and second derivative by substituting both coordinates of a given point. (Note: If all that is needed is the numerical value of the derivative then the substitution is often easier if done before solving for dy/dx or d^2y/dx^2 and as usual the arithmetic need not be done.)
- Analyze the derivative to determine where the relation has horizontal and/or vertical tangents.
- Write and work with lines tangent to the relation.
- Find extreme values. It may also be necessary to show that the point where the derivative is zero is actually on the graph and to justify the answer.

Related Rates
What students should know how to do:

- Set up and solve related rate problems.
- Be familiar with the standard type of related rate situations, but also be able to adapt to different contexts.
- Know how to differentiate with respect to *time*, that is find dy/dt even if there is no time variable in the given equations, using any of the differentiation techniques.
- Interpret the answer in the context of the problem.
- Unit analysis

NOTES: