Volumes, Books, and Pancakes

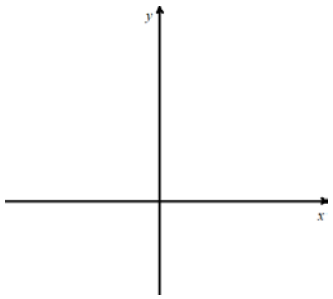
AP Calculus

Name _____

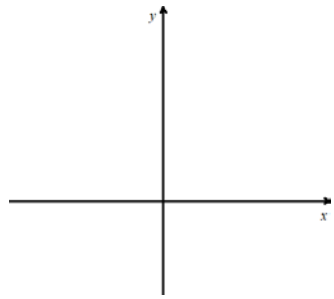
Date _____ Period _____

Sketch the region bounded by $y = x^2$, the x-axis, and the line $x = 2$. Then, revolve the region about each of the indicates lines. Sketch each situation, write the integral, and solve it using a calculator.

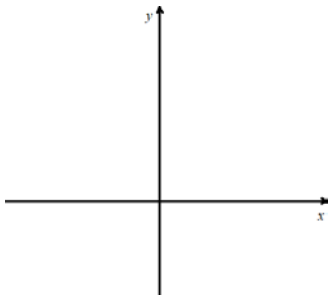
1. **About the x-axis**



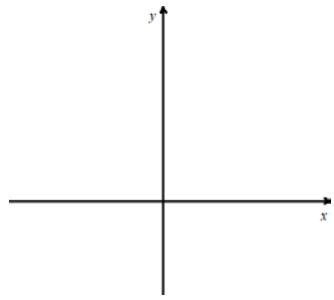
2. **About the line $x = 2$**



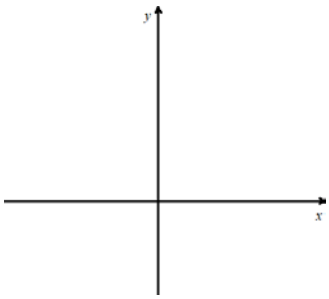
3. **About the y-axis**



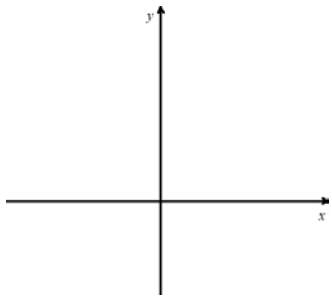
4. **About the line $x = 3$**



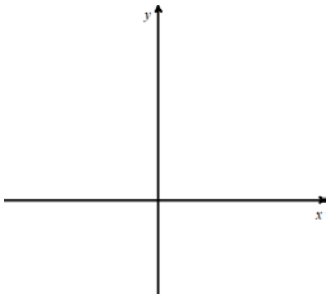
5. About the line $y = 5$



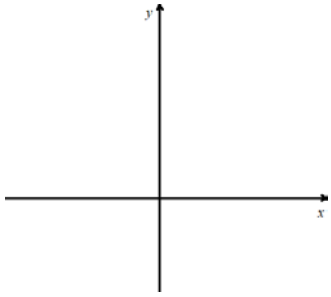
6. About the line $y = -1$



7. About the line $x = -1$



8. Region bounded by $y = x^2$ and $y = \sqrt{x}$ revolved about the x–axis.



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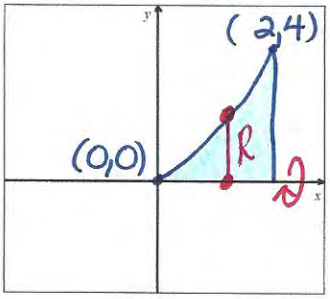
AP Calculus

Name Key

Date _____ Period _____

Sketch the region bounded by $y = x^2$, the x-axis, and the line $x = 2$. Then, revolve the region about each of the indicates lines. Sketch each situation, write the integral, and solve it using a calculator.

1. About the x-axis



disk "books"

$a=0$ $b=2$

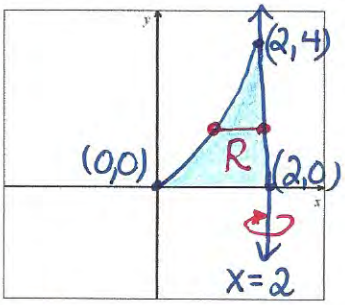
$V = \pi \int_0^2 (x^2)^2 dx$

$= \pi \int_0^2 x^4 dx$

$= \pi \left[\frac{1}{5}x^5\right]_0^2$

$V = \frac{32\pi}{5}$

2. About the line $x = 2$



disk/"pancakes"

$c=0$ $d=4$

radius: $2-\sqrt{y}$

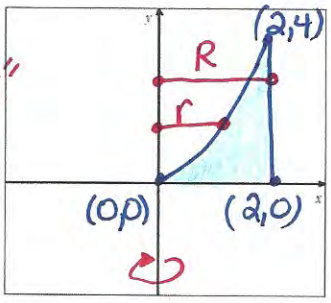
$V = \pi \int_0^4 (2-\sqrt{y})^2 dy$

$V = \pi \int_0^4 [4 - 4y^{1/2} + y] dy$

$V = \pi \left[4y - \frac{8}{3}y^{3/2} + \frac{1}{2}y^2\right]_0^4$

$V = \frac{8\pi}{3}$

3. About the y-axis



washer/"pancakes"

$c=0$ $d=4$

R: $x=2$

r: $x=\sqrt{y}$

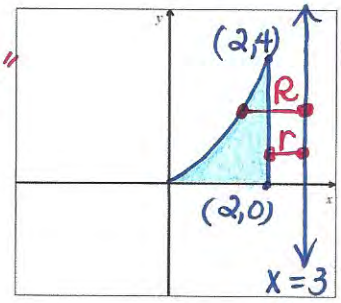
$V = \pi \int_0^4 2^2 - (\sqrt{y})^2 dy$

$= \pi \int_0^4 (4-y) dy$

$= \pi \left[4y - \frac{1}{2}y^2\right]_0^4$

$V = 8\pi$

4. About the line $x = 3$



washer/"pancakes"

$c=0$ $d=4$

R: $3-\sqrt{y}$

r: $3-2=1$

$V = \pi \int_0^4 [(3-\sqrt{y})^2 - 1^2] dy$

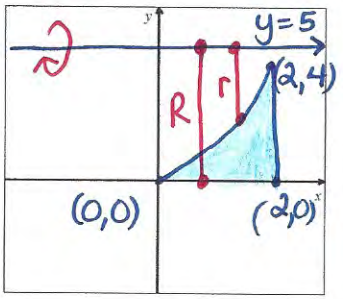
$V = \pi \int_0^4 (9 - 6y^{1/2} + y - 1) dy$

$= \pi \int_0^4 8 - 6y^{1/2} + y \, dy$

$= \pi \left[8y - 4y^{3/2} + \frac{1}{2}y^2\right]_0^4$

$V = 8\pi$

5. About the line $y = 5$



Washer / "books"

$a = 0 \quad b = 2$

R: $5 - 0$

r: $5 - x^2$

$$V = \pi \int_0^2 5^2 - (5 - x^2)^2 \, dx$$

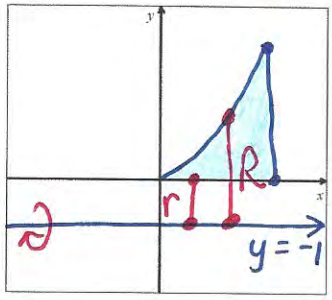
$$V = \pi \int_0^2 25 - (25 - 10x^2 + x^4) \, dx$$

$$= \pi \int_0^2 10x^2 - x^4 \, dx$$

$$= \pi \left[\frac{10x^3}{3} - \frac{1}{5}x^5 \right]_0^2$$

$$V = \frac{304\pi}{15}$$

6. About the line $y = -1$



Washer / "books"

$a = 0 \quad b = 2$

R: $x^2 - (-1)$

r: $0 - (-1)$

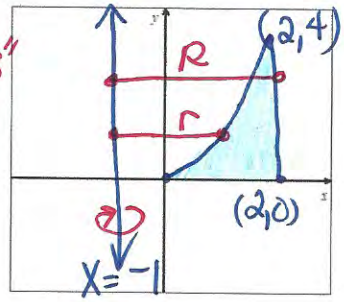
$$V = \pi \int_0^2 (x^2 + 1)^2 - (1)^2 \, dx$$

$$= \pi \int_0^2 (x^4 + 2x^2 + 1 - 1) \, dx$$

$$= \pi \left[\frac{1}{5}x^5 + \frac{2}{3}x^3 \right]_0^2$$

$$V = \frac{176\pi}{15}$$

7. About the line $x = -1$



Washer / "pancakes"

$c = 0 \quad d = 4$

R: $2 - (-1)$

r: $\sqrt{y} - (-1)$

$$V = \pi \int_0^4 (3)^2 - (\sqrt{y} + 1)^2 \, dy$$

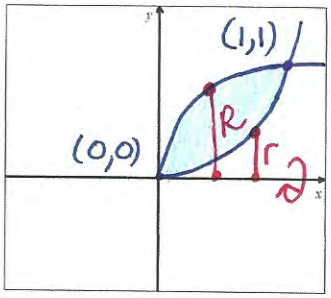
$$= \pi \int_0^4 9 - (y + 2y^{1/2} + 1) \, dy$$

$$= \pi \int_0^4 8 - y - 2y^{1/2} \, dy$$

$$= \pi \left[8y - \frac{1}{2}y^2 - \frac{4}{3}y^{3/2} \right]_0^4$$

$$V = \frac{40\pi}{3}$$

8. Region bounded by $y = x^2$ and $y = \sqrt{x}$ revolved about the x-axis.



Washer / "books"

$a = 0 \quad b = 1$

R: $\sqrt{x}$

r: x^2

$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$$

$$= \pi \int_0^1 x - x^4 \, dx$$

$$= \pi \left[\frac{1}{2}x^2 - \frac{1}{5}x^5 \right]_0^1$$

$$V = \frac{3\pi}{10}$$