

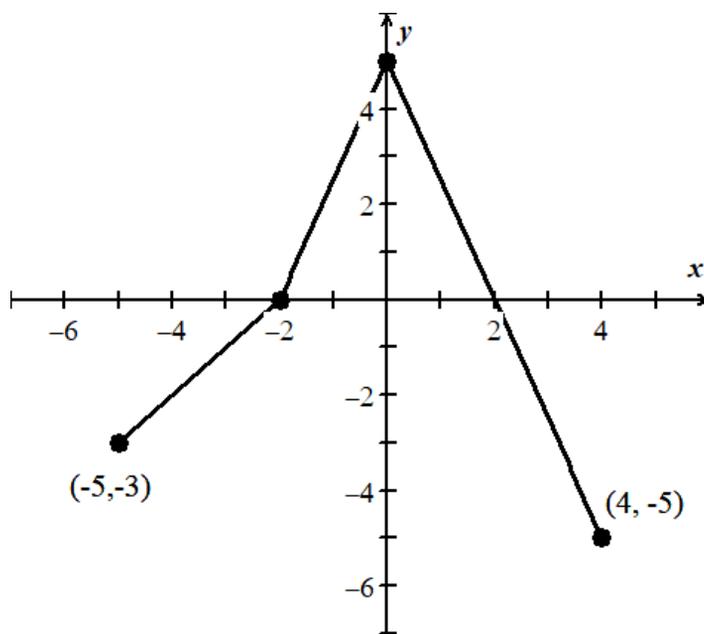
Graph of f

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by

$$g(x) = \int_{-2}^x f(t) dt$$

- Find the $g(2)$, $g'(2)$ and $g''(2)$.
- For what values of x in the open interval $-5 < x < 4$ is g decreasing? Explain your reasoning.
- For what values of x in the open interval $-5 < x < 4$ is g concave up? Explain your reasoning.
- Find all values of x on the open interval $-5 < x < 4$ for which the function g has a local minimum. Justify your answer.
- On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- Does the graph of g have a point of inflection at $x = 0$? Justify your answer.
- Find the absolute minimum value and the absolute maximum value of g on the interval $[-5, 4]$. Justify your answers.
- For $[-5, 4]$, find all intervals for which $g(x) \leq 0$.

Graph Analysis & Justification Practice Key



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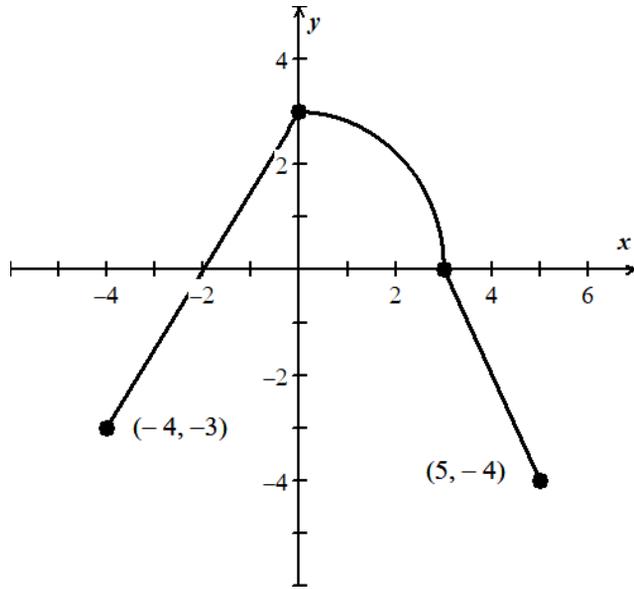
$$g(x) = \int_{-2}^x f(t) dt$$

KEY:

- A) $g(2) = \int_{-2}^2 f(t) dt = \frac{1}{2}(4)(5) = 10$ $g'(2) = 0$ and $g''(2) = -\frac{5}{2}$
- B) G decreases where $g'(x) = f(x)$ is negative. G decreases $-5 < x < -2$ and $2 < x < 4$.
- C) G is concave down where $g'(x) = f(x)$ is decreasing. f is decreasing on $0 < x < 4$, so G is concave down here.
- D) G has a local minimum when f changes from negative to positive, so at $x = -2$, G has a local minimum.
- E) The graph of G will be increasing and concave down where $g'(x) = f(x)$ is positive and decreasing. This happens on $0 < x < 2$.
- F) G has a point of inflection where $f'(x)$ is zero or dne. Candidates are $x = -2$ and $x = 0$. Also, where $g'(x) = f(x)$ changes from increasing to decreasing. So $g(x)$ has a point of inflection at $x = 0$ because f changes from increasing to decreasing here.
- G) The graph of G has an absolute max or absolute min where the derivative changes signs and where $g' = f$ is zero or dne. By the candidates test, $G(-5) = 4.5$, $G(-2) = 0$, $g(2) = 10$, $g(4) = 5$. Absolute max value is 10 absolute min value is 0.
- H) $G(x) \geq 0$ on the entire interval and is never negative.

Graph Analysis &
More Justification Practice

Name _____

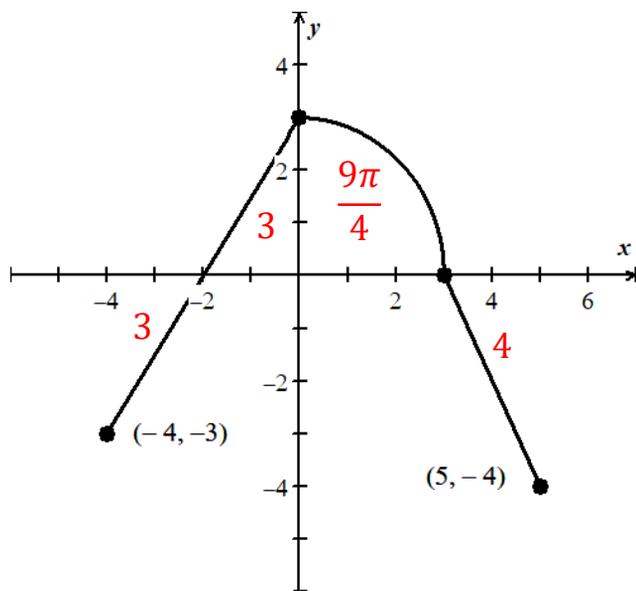


Graph of f'

Let f be a function defined on the closed interval $[-4, 5]$ with $f(0) = -2$. The graph of f' , the derivative of f , consists of two line segments and a quarter circle, as shown above.

- A) On what intervals, if any, is f increasing? Justify your answer.
- B) For $-4 \leq x \leq 5$, find the value of x at which f has an absolute maximum. Justify your answer.
- C) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-4 < x < 5$. Justify your answer.
- D) Find an equation for the line tangent to the graph of f at the point $(0, -2)$.
- E) Find $f(-4)$ and $f(5)$. Show work that leads to your answers.

Graph Analysis & More Justification Practice Key



Graph of f'

Let f be a function defined on the closed interval $[-4, 5]$ with $f(0) = -2$. The graph of f' , the derivative of f , consists of two line segments and a quarter circle, as shown above.

A) On what intervals, if any, is f increasing? Justify your answer.

f is increasing on $-2 \leq x \leq 3$ because $f'(x) > 0$ on this interval.

B) For $-4 \leq x \leq 5$, find the value of x at which f has an absolute maximum. Justify your answer.

On $-4 \leq x \leq 5$, f would obtain a maximum where $f(x)$ changes from positive to negative. This occurs at $x = 3$.

C) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-4 < x < 5$. Justify your answer.

Where $f'(x)$ changes from increasing to decreasing there is a point of inflection so, at $x = 0$.

D) Find an equation for the line tangent to the graph of f at the point $(0, -2)$.

$f'(0) = 3$ and $(0, -2)$ Tangent line: $y = 3x - 2$

E) Find $f(-4)$ and $f(5)$. Show work that leads to your answers.

$f(-4)$ is: IC + $\int_{-4}^0 f'(t) dt$: $-2 + \text{area}(-3 + 3) = -2$

$f(5)$ is: IC + $\int_0^5 f'(t) dt$: $-2 + \text{area}\left(\frac{9\pi}{4} - 4\right) = -6 + \frac{9\pi}{4}$